00232

The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme
Final Examination-2014/2015
APU 2144/APE4144- Applied Linear Algebra and Differential Equations
APPLIED MATHEMATICS-LEVEL 04

Duration: Two Hours.

Date: 04.11.2015	**Time: 09.30 a.m. - 11.30 a.m.**

Answer FOUR questions only.

1. (a) Find a value of k so that the matrix A where

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & 1 \\ 2 & 3 & k \end{bmatrix}$$

is singular.

(b) Find the rank of the matrix B where $B = \begin{bmatrix} 2 & -2 & 4 & 3 \\ 1 & -1 & 2 & 2 \\ 2 & -2 & 1 & 2 \\ 1 & -1 & 0 & 1 \end{bmatrix}$.

(c) Find the values of a and b for which the system of equations

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + az = b.$$

has (i) no solution

(ii) a unique solution

(iii) an infinitely many solutions.

2. (a) Let A, B and C be three matrices. Prove each of the following:

(i) If B and C are two inverses of A, then $B = C$.

(ii) $(AB)^{-1} = B^{-1}A^{-1}$.

(iii) $\left(A^{-1}\right)^T = \left(A^T\right)^{-1}$

(iv) $\left(A^{-1}\right)^{-1} = A$.

(b) Find the orthogonal transformation which transforms the quadratic form

$$2\left(x_1x_2 + x_2x_3 + x_3x_1\right)$$

to canonical form (or "sum of squares form").

3. Find the general solutions of each of the systems of simultaneous differential equations, given below. Here, the dots signify a standard notation involving the parameter t.

(a)
$$\begin{aligned} \dot{x}_1 &= x_1 - x_2 - x_3 \\ \dot{x}_2 &= x_1 + 3x_2 + x_3 \\ \dot{x}_3 &= -3x_1 + x_2 - x_3 \end{aligned}$$

(b)
$$\begin{aligned} \dot{x}_1 &= -2x_1 + 10x_2 + 48 \\ \dot{x}_2 &= 10x_1 - 2x_2 + 91e^t \end{aligned}$$

(c)
$$\begin{aligned} \ddot{x} &= 8x - 5y \\ \ddot{y} &= 10x - 7y \end{aligned}$$

4. (a) Find a sinusoidal particular solution for the following system of partial differential equations:

$$\begin{aligned} \ddot{x}_1 + 2\ddot{x}_2 + \dot{x}_1 + x_1 - 3x_2 &= \sin t \\ 3\ddot{x}_1 + \ddot{x}_2 + 2\dot{x}_2 + 2x_1 + x_2 &= \cos t - 2\sin t. \end{aligned}$$

(b) Find the general solution of each of the following simultaneous partial differential equations:

(i) $\dfrac{\partial u}{\partial x} = 6x^2, \quad \dfrac{\partial u}{\partial y} = 12y^2.$

(ii) $\dfrac{\partial u}{\partial x} = 4xy + a\cos ax, \quad \dfrac{\partial u}{\partial y} = 2x^2 + 9e^{3y}.$

(c) Find the general solution of the following differential equation:

$$x^2 \frac{d^2 y}{dx^2} - 5x\frac{dy}{dx} + 8y = 2x^3.$$

5. Consider the partial differential equation $x^2 \dfrac{\partial u}{\partial x} + y^2 \dfrac{\partial u}{\partial y} - x^2 u = 1 \quad (x \neq 0, y \neq 0).$

(a) With the transformation $\zeta = x$ and $\phi = \dfrac{1}{y} - \dfrac{1}{x}$ find using the chain rule, expressions for $\dfrac{\partial u}{\partial x}$ and $\dfrac{\partial u}{\partial y}$ in terms of $\dfrac{\partial u}{\partial \zeta}$ and $\dfrac{\partial u}{\partial \phi}$.

(b) Hence, simplify the partial differential equation to the form $\dfrac{\partial u}{\partial \zeta} - u = \dfrac{1}{\zeta^2}$.

(c) Solve this equation for u(ζ, ϕ), and hence find the general solution of the original equation in terms of x and y.

6. Show that the general solution of the equation $\dfrac{\partial^2 u}{\partial x^2} - 4x^2 \dfrac{\partial^2 u}{\partial y^2} - \dfrac{1}{x}\dfrac{\partial u}{\partial x} = 0, \quad (x \neq 0)$

may be found by reducing it to the standard form $\dfrac{\partial^2 u}{\partial \zeta \partial \phi} = 0$, where $\zeta = x^2 - y$ and $\phi = x^2 + y$. Use the method of characteristics to derive expressions for ζ and ϕ.