The Open University of Sri Lanka

Faculty of Natural Sciences, Department of Chemistry

B. Sc Degree Programme 2015/2016

PSE 3117 Mathematics for Chemistry and Biology

Assignment Test I - (1 hour)

Date - 5th April 2016	**Time - 4.15 p.m to 5.15 p.m**

Registration Number:

Total Marks = 230

Those scoring 200 or more would be deemed to have scored 100%;

pro-rata marks will be awarded to others

- **This paper consists of four (4) Structured Questions** (answer **only** in the space provide)

  මෙම ප්‍රශ්ණ පත්‍රයේ ව්‍යුහගත ප්‍රශ්ණ 4 ක් අන්තර්ගත වේ. සපයා ඇති ඉඩ ප්‍රමාණයේ පමණක් පිළිතුරු ලියන්න.

  இவ்வினாத்தாள் 4 கட்டமைப்பு வினாக்களை கொண்டது (தரப்பட்டுள்ள இடைவெளியில் மட்டும் விடையளிக்க).

- The use of electronic calculator is **not** permitted.

  ගණක යන්ත්‍ර භාවිතා කළ නොහැක.

  நெறிப்படுத்தப்படாத கணினியின் பயன்பாடுஅனுமதிக்கப்பட்டுள்ளது.

- Switch off **mobile phones and leave them outside.**

  ජංගම දුරකථන ක්‍රියා විරහිත කර ශාලාවෙන් පිටත තබන්න.

  கையடக்கத் தொலைபேசிகளை நிறுத்திவிடவும்.

- Write down, clearly, your **NAME and ADDRESS** in the **LAST** page

  අවසාන පිටුවෙහි ඔබගේ නම සහ ලිපිනය පැහැදිලිව ලියන්න.

  விடைத்தாளின் **பின் பக்கத்தில்** உமது பெயரையும் விலாசத்தையும் தெளிவாக எழுதுக.

1.

(a) Resolve into partial fraction. හින්න භාගවලට වෙන් කරන්න.
பகுதிப் பின்னங்களைத் தீர்க்க.

(i) $\dfrac{x^2-4x+7}{(x-1)(x^2+3x-10)}$

(ii) $\dfrac{(x+16)}{(2x+3)(x-2)}$

(20 marks)

(b) Evaluate the following with important steps (**without the aid of a calculator or log tables**)
ගණක යන්ත්‍ර හෝ ලඝු ගණක වගු භාවිතා නොකර පහත දැක්වෙන දැනි අගය සොයන්න.
(පියවර සහිතව) பின்வருவனவற்றை கணிக்குக (கணினி, மடக்கையின் உதவியின்றி)

$$\frac{[log_4 64 + log_2 16]}{log_{10} 100}\left[log_5 125 + 2log_3 \sqrt[3]{27}\right]$$

(20 marks)

(c) Evaluate using Binomial Theorem/Expansion (steps are important)
ද්විපද ප්‍රසාරණයේ ඔයිනෝමියල් සිද්ධාන්තය (Binomial Theorem) අනුව ගණනය කරන්න. (පියවර වැදගත් වේ.)
ஈருறுப்புதேற்றத்தை/விரிவைப் பயன்படுத்தி கணிக்குக(படிமுறைகள் முக்கியம்)

$(a-2)^5$

(10 marks)

(d) Find the values of Limit. සීමාව සොයන්න.
எல்லைப் பெறுமானங்களைதருக

(i) $\lim_{x \to 2} \frac{x^2+4x-12}{x-2}$

(ii) $\lim_{h \to 0} \frac{(h+3)^2-9}{h}$

(iii) $\lim_{x\to\infty} \frac{x^2+x+1}{(3x+2)^2}$

(iv) $\lim_{x\to\infty} \frac{x+2}{x^2-2}$

(40 marks)

2. (a) Solve: විසඳන්න. தீர்க்குக.

$$(a + 2)(a - 6) = (a - 2)(a - 8)$$

(10 marks)

(b) Express the following in the complex number form, a + ib:

පහත දැක්වෙන දෑ a + ib ආකාරයේ සංකීර්ණ සංඛ්‍යාවක් ලෙස දක්වන්න

பின்வருவனவற்றை சிக்கல் எண் வடிவில் விளக்குக.

$$\frac{(3-2i)(2+i)}{(3-i)}$$

(10 marks)

(c) Prove that ඔව පෙන්වන්න.
நிறுவுக.

(i) $\frac{Sin^4\theta - Cos^4\theta}{Sin^2\theta - Cos^2\theta} = 1$

(ii) $\sec\theta - \tan\theta = \frac{\cos\theta}{1+\sin\theta}$

(20 marks)

3. (a) Differentiate the function $y = bx^2$ by using first principle.
$y = bx^2$ ශ්‍රිතය ප්‍රථම මූලධර්ම ක්‍රමයෙන් අවකලනය කරන්න.

முதல் கொள்கையைப் பயன்படுத்தி $y = bx^2$ என்றசார்பைவகையிடுக.

(10 marks)

(b) Given that $y = x^2.e^{2x} - 2e^{2x}$, show that $\frac{dy}{dx} = 2y + 2x.e^{2x}$

$y = x^2.e^{2x} - 2e^{2x}$, ද ඇති විට $\frac{dy}{dx} = 2y + 2x.e^{2x}$ බව පෙන්වන්න.

$y = x^2.e^{2x} - 2e^{2x}$எனும் சார்பு தரப்பட்டுள்ளது. $\frac{dy}{dx} = 2y + 2x.e^{2x}$எனக் காட்டுக.

(10 marks)

(c) Obtain the **first differential** of the following:

පහත දැක්වෙන දෑ හි **පළමු අවකලය** ලබාගන්න

பின்வரும் கோவையின் **முதலாம் வகையீட்டைப்** பெறுக.

(i) $V = (2x + 1)^3(2x^2 + 3x + 1)$

(ii) $Y = \dfrac{\text{Sin } 2\theta}{\text{Cos } 3\theta}$

(iii) $U = 3x^2e^{2x}$

(iv) $Y = x \log 2x$

(v) $Y = \frac{X^{1/2}+2}{X^{3/2}}$

(50 marks)

(4) The function $y = K[x + x^2 - x^3]$ (where K is arbitrary constants) has two turning/stationary points, The point (1,2) lie on the curve .

K අහිමත නියතයක් වන $y = K[x + x^2 - x^3]$ යන ශ්‍රිතයට වර්තන ස්ථාවර ලක්ෂ්‍ය දෙකක් ඇත. මෙම වක්‍රය (1,2) ලක්ෂ්‍යය හරහා ගමන් කරයි නම්

$y = K[x + x^2 - x^3]$ என்னும் சார்பு இரு திரும்பற் புள்ளிகளை உடையது. இவற்றுள் ஒன்று (1,2) ஆகும்.

(i) Determine the value of ***K***. ***K*** හි අගය සොයන්න.

K இன் பெறுமானத்தை தீர்மானிக்க.

(ii) Determine the turning points. වර්තන ලක්ෂ්‍යයන් සොයන්න.

மற்றைய திரும்பற் புள்ளியைத் தீர்மானிக்க.

(iii) Identify these turning points as maxima, minima or points of inflexion.
වර්තන ලක්ෂයයන් උපරිම අවම හෝ නතිවර්තන දැයි හඳුනා ගන්න
இத் திரும்பற் புள்ளிகள் உயர்வா, இழிவா அல்லது விபத்தைப் புள்ளியா என இனங்காண்க.

(30 marks)

Registration No

Name : .....

Address: .....

.....

.....

.....

The Open University of Sri Lanka

Faculty of Natural Sciences, Department of Chemistry

B. Sc degree programme 2015/2016

PSE 3117 Mathematics for Chemistry and Biology

Assignment Test I –Answer Guide

1.(a)

(i) $\frac{x^2-4x+7}{(x-1)(x^2+3x-10)} = \frac{A}{x-1} + \frac{B}{x+5} + \frac{C}{x-2}$

$= A(x+5)(x-2) + B(x-1)(x-2) + C(x-1)(x+5)$

x=1, $4 = -6A$, $A=-2/3$

x=2 $3= 7C$, $C= 3/7$

x=-5 $52= 42B$, $B= 52/42$, $= 26/21$

$= \frac{3}{7(x-2)} - \frac{2}{3(x-1)} - \frac{26}{21(x+5)}$

(ii) $\frac{(x+16)}{(2x+3)(x-2)} = \frac{A}{2x+3} + \frac{B}{x-2}$

$x+16 = A(x-2) + B(2x+3)$

x=2, $18 = 7B$, $B = 18/7$

x= -3/2 $29/2 = A(-7/2)$, $A = -29/7$

$= \frac{18}{7(x-2)} - \frac{29}{7(2x+3)}$

(b)

$$\frac{[\log_4 64 + \log_2 16]}{\log_{10} 100}\left[\log_5 125 + 2\log_3 \sqrt[3]{27}\right]$$

$$\frac{[log_4 4^3 + log_2 2^4]}{log_{10} 10^2}\left[log_5 5^3 + 2log_3 (3^3)^{1/3}\right]$$

$$\frac{[3+4]\ (3+2)}{2} = 35/2$$

(c) $(a-2)^5 = {}^5C_5\, a^5 - {}^5C_4\, a^4 * 2 + {}^5C_3\, a^3 * 2^2 - {}^5C_2\, a^2 * 2^3 + {}^5C_1\, a * 2^4 - {}^5C_0\, a^0 * 2^5$

$= a^5 - 10a^4 + 40a^3 - 80a^2 + 80a - 32$

(d) (i) $\lim_{x\to 2}\frac{x^2+4x-12}{x-2} = \lim_{x\to 2}\frac{(x-2)(x+6)}{(x-2)} = \lim_{x\to 2}\frac{(x-2)(x+6)}{(x-2)}$

$= \lim_{x\to 2} x+6 = 8$

(ii) $\lim_{h\to 0}\frac{(h+3)^2-9}{h} = \lim_{h\to 0}\frac{(h+3-3)(h+3+3)}{h} = 6$

(iii) $\lim_{x\to\infty}\frac{x^2+x+1}{(3x+2)^2} = \lim_{x\to\infty}\frac{x^2+x+1}{9x^2+12x+4} = \lim_{x\to\infty}\frac{x^2/x^2+x/x^2+1/x^2}{9x^2/x^2+12x/x^2+4/x^2} = \frac{1}{9}$

(iv) $\lim_{x\to\infty}\frac{x+2}{x^2-2} = \lim_{x\to\infty}\frac{x/x^2+2/x^2}{x^2/x^2-2/x^2} = 0$

2. $(a+2)(a-6) = (a-2)(a-8), \Longrightarrow a^2 - 4a - 12 = a^2 - 10a + 16$

$6a = 28$

$a = 28/6$

(b) $\frac{(3-2i)(2+i)}{(3-i)} = \frac{(3-2i)(2+i)(3+i)}{(3-i)(3+i)} = \frac{(6-i-2i^2)(3+i)}{(9-i^2)} = \frac{(8-i)(3+i)}{(10)} = \frac{(24+5i-i^2)}{(10)}$

$= \frac{5(5+i)}{10} = \frac{5+i}{2}$

(c) (i) $\frac{Sin^4\theta - Cos^4\theta}{Sin^2\theta - Cos^2\theta} = 1$

$\frac{(Sin^2\theta - Cos^2\theta)((Sin^2\theta + Cos^2\theta)}{(Sin^2\theta - Cos^2\theta)} = (Sin^2\theta + Cos^2\theta) = 1$

(ii) $\sec\theta - \tan\theta = \frac{\cos\theta}{1+\sin\theta}$

$\frac{1}{Cos\theta} - \frac{Sin\theta}{Cos\theta} = \frac{(1-Sin\theta)}{Cos\theta}\frac{(1+Sin\theta)}{(1+Sin\theta)} = \frac{1-Sin^2\theta}{(Cos\,\theta)(1+Sin\theta)} = \frac{Cos^2\theta}{(Cos\,\theta)(1+Sin\theta)}$

$= \frac{Cos\,\theta}{1+Sin\theta}$

3. (a) Consider the δx and δy as x and y

$y + \delta y = b(x+\delta x)^2$

$\delta y = b(x+\delta x)^2 - y = b(x+\delta x)^2 - bx^2 = b(x+\delta x - x)(x+\delta x + x)$

$\lim_{x\to 0}\frac{\delta y}{\delta x} = \lim_{x\to 0} b(2x+\delta x)$

$\frac{dy}{dx} = 2bx$

(b) $Y = x^2e^{2x} - 2e^{2x}$

$2y = 2(x^2e^{2x} - 2e^{2x})$, $\frac{dy}{dx} = 2xe^{2x} + x^2e^{2x} * 2 - 2e^{2x} * 2$

$= 2xe^{2x} + [\,2(x^2e^{2x} - 2e^{2x})] = 2y + 2xe^{2x}$

(c) Obtain the **first differential** of the following:

(i) $V = (2x+1)^3(2x^2+3x+1)$

$$\frac{dv}{dx} = (2x+1)^3\frac{d}{dx}(2x^2+3x+1) + (2x^2+3x+1)\frac{d}{dx}(2x+1)^3$$

$$\frac{dv}{dx} = (2x+1)^3 * (4x+3) + (2x^2+3x+1)*3*(2x+1)^2 * 2$$

$$= (2x+1)^3 * (4x+3) + 6(2x^2+3x+1)*(2x+1)^2$$

(ii) $Y = \frac{\text{Sin } 2\theta}{\text{Cos } 3\theta}$

$$\frac{dy}{d\theta} = \frac{\cos 3\theta\frac{d(\sin 2\theta)}{d\theta} - \sin 2\theta\frac{d(\cos 3\theta)}{d\theta}}{(\cos 3\theta)^2} = \frac{\cos 3\theta*\cos 2\theta*2 - \sin 2\theta*-\sin 3\theta*3}{(\cos 3\theta)^2}$$

$$\frac{dy}{d\theta} = \frac{2\cos 3\theta*\cos 2\theta + 3\sin 2\theta*\sin 3\theta}{(\cos 3\theta)^2}$$

(iii) $U = 3x^2e^{2x}$

$$\frac{dU}{dx} = 3x^2\frac{d}{dx}e^{2x} + e^{2x}\frac{d(3x^2)}{dx} = 3x^2*e^{2x} * 2 + e^{2x} * 6x$$

$$= 6x * e^{2x}(x+1)$$

(iv) $Y = x \log 2x$

$$\frac{dy}{dx} = x\frac{d(\log 2x)}{dx} + \log 2x\frac{d(x)}{dx} = x * \frac{1}{2x} * 2 + \log 2x * 1$$

$$= 1 + \log 2x$$

(v) $Y = \frac{X^{\frac{1}{2}}+2}{X^{\frac{3}{2}}}$

$$\frac{dy}{dx} = \frac{x^{3/2}\frac{d\left(x^{1/2}+2\right)}{dx} - \left(x^{1/2}+2\right)\frac{d\left(x^{3/2}\right)}{dx}}{(x^{3/2})^2} = \frac{x^{3/2} * \frac{1}{2}x^{-1/2} - \left(x^{1/2}+2\right)\frac{3}{2}x^{1/2}}{(x^{3/2})^2}$$

$$\frac{dy}{dx} = \frac{\frac{1}{2}x - \frac{3}{2}x - 3x^{1/2}}{x^3} = \frac{-x-3x^{1/2}}{x^3}$$

(4) (i) $Y = K(x + x^2 - x^3)$

$2 = K(1+1-1)$

$K = 2$

(iii) $Y = 2(x+x^2-x^3)$

$$\frac{dy}{dx} = 2(1 + 2x - 3x^2)$$

$\frac{dy}{dx} = 0$ for turning points.

$2 \neq 0 \quad 1 + 2x - 3x^2 = 0$

$(1 - x)(1 + 3x) = 0$

$1 - x = 0$ or $1 + 3x = 0$

$x = 1 \; or \; x = -1/3$

(iii) $\frac{d^2y}{dx^2} = 2(2 - 6x)$

$x = 1 \qquad \frac{d^2y}{dx^2} = -8 < 0$

Therefore x=1 turning point is minima

x= -1/3 $\qquad \frac{d^2y}{dx^2} = 8 > 0$

Therefore x= -1/3 turning point is maxima