**The Open University of Sri Lanka**

**B.Sc. Degree Programme-Level 5**

**Final Examination-2016**

**Mathematical Physics-PHU 3148/PHE 5148**

**Duration:Two(02) hours.**

**Answer FOUR(04) questions only.**

**Date:23rd July 2016** **Time: 9.30 a.m. to 11.30 a.m.**

(1) (i) Sate the binomial expansion for $(a+b)^n$. Here n is integer and $a, b$ are constants.

(ii) Obtain the expansion of $(1+x)^4$ using the above expansion.

(iii) Check whether expansion is true when substituting $x = 2$.

(iv) In the special relativity, the factor $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ is very important, where $v$ is any possible velocity of a particle and $c$ is the velocity of light.

Considering inequality $\frac{v}{c} \ll 1$,

Prove that
$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \approx 1 + \left(-\frac{1}{2}\right)\left(-\frac{v^2}{c^2}\right) = 1 + \frac{v^2}{2c^2}.$$

(2) (i) State the necessary and sufficient condition for the differential equation $M(x,y)\,dx + N(x,y)\,dy = 0$ to be exact.

Solve $\left(1 + e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0$

(ii) In a special particles field, the amount A of active particle grows at a rate proportional to the amount present. If the original amount $A_0$ doubles in 2 hours. How long does it take for the original amount to triple?

(3) (a) Find the Fourier series of the function f(x) = x in the range $-\pi \leq x \leq \pi$. Hence show that,

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$$

(b) Find the Fourier transform of the function given below.

$$f(x) = \begin{cases} \frac{1}{2} & -3 < x < 3 \\ 0 & otherwise \end{cases}$$

(4). (a) The position vector of a particle at time t in Cartesian coordinates is given by $r(t) = 2t^2 i + (3t-2) j + (3t^2 - 1) k$. Find the speed of the particle at t = 1 and the component of its acceleration in the direction s = i+ 2j + k.

(b) A small particle of mass *m* orbits a much larger mass *M* centred at the origin *O*. According to Newtons's law of gravitation, the position vector *r* of the small mass obeys the differential equation

$$m\frac{d^2 r}{dt^2} = -\frac{GMm}{r^2}\hat{r}$$

Show that the vector $r \times \frac{dr}{dt}$ is a constant of the motion.

(c) If $\bar{a} = 2i - j + 3k,\ \bar{b} = 3i + 2j + k,\ \bar{c} = i + \rho j + 4k$ are coplanar $\left(\text{i.e.} \left|\bar{a}.\left(\bar{b} \times \bar{c}\right)\right| = 0\right)$, find the value of $\rho$.

(5). (A) Define the Laplace transformation.

(B) (a) If $f(t)$ is continuous for $t > 0$ and of exponential order as $t \to \alpha$, while $f'(t)$ is sectionally continuous for $t > 0$ and if $L\{f(t)\} = F(s)$, show that $L\{f'(t)\} = sF(s) - f(0)$

(b) Find the Laplace transform of $f'(t)$ when

(i) $f(t) = e^{8t}$

(ii) $f(t) = \sin at$ where "$a$" is a constant.

(iii) $f(t) = 1$

(C) Solve the following differential equation for *f(t)* by using the Laplace transformation.

$$f''(t) + f'(t) + 2f(t) = Sin\ t$$

$$f(t) = 0; \ f'(t) = 0$$

(6). (a) Solve the following linear equations by the matrix method

$x + y + z = 9$

2x+5y+7z =52

2x+y-z = 0

(b) Determine the eigen values of the following matrix.

$$\begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$$