THE OPEN UNIVERSITY OF SRI LANKA
B.Sc. DEGREE PROGRAMME
PURE MATHEMATICS -LEVEL 05
PUU3244/PUE5244 — Number Theory & Polynomials
NO BOOK TEST-2015/2016

DURATION: ONE AND HALF (1 ½) HOURS

Date:- 06.11. 2016 **Time:- 10:30a.m. –12:00noon.**

ANSWER ALL QUESTIONS.

(01). (a) Define each of the followings:

(i) Greatest Common Divisor.

(ii) Least Common Multiple.

(iii) Pair wise relatively prime.

(iv) Mutually relatively prime.

(b) If $a,b,c \in \mathbb{Z}\setminus\{0\}$ prove that $(a,b,c) = ((a,b),c)$.

(c) Compute $d = (1044,1116,1470)$ and express it in the form

$$d = 1044a + 1116b + 1470c.$$

(02). (a) Prove each of the following:

(i) If $a + c \equiv b + c (\text{mod}\, m)$ then
$a \equiv b (\text{mod}\, m)$.

(ii) If $ac \equiv bc (\text{mod}\, m)$ and $(c,m) = 1$ then
$a \equiv b (\text{mod}\, m)$.

(b) If $n \in \mathbb{N}$ prove that

(i) $10^n \equiv 1 (\text{mod}\, m)$.

(ii) $6^n \equiv 6 (\text{mod}\, 10)$.

(c) State Eisentein's irreducibility criteria.

(d) Determine whether the following polynomials are irreducible over $\mathbb{Q}[x]$.

(i) $f(x) = 8x^3 + 6x^2 - 9x + 24$

(ii) $f(x) = x^3 + 3x^2 - 8$

(03). (a) Find all rational roots of the polynomial $6x^4 + 23x^3 + 28x^2 + 13x + 2$ over $\mathbb{Q}$.

(Hint :Let $f(x) = \sum_{i=0}^{n} a_i x^i \in \mathbb{Z}[x]$ and $n \geq 1$. If $\alpha \in \mathbb{Q}$ is a zero of $f(x)$ and $\alpha = \frac{r}{s}$ with $(r, s) = 1$, then $r \mid a_0$ and $s \mid a_n$.)

(b) If $f(x) = x^4 + 4x^3 + 3x^2 + x + 1$ and $g(x) = 2x^3 + x^2 - x - 3$ are polynomials over $\mathbb{Z}_5[x]$. Find the greatest common divisor d of $f(x)$ and $g(x)$ and express it in the form $d = fu + gv$ with $u, v \in \mathbb{Z}_5[x]$.