The Open University of Sri Lanka

B.Sc. /B.Ed. Degree Programme

Applied Mathematics – Level 05

APU3143/APE5143 – Mathematical Methods

No Book Test (NBT) – 2016/2017

DURATION: ONE HOUR

Date: 21.05.2017.   **Time: 14:30h – 15:30h**

ANSWER **ALL** QUESTIONS.

1. (i) The Gamma function denoted by $\Gamma(p)$ corresponding to the parameter p is defined by

   the improper integral $\Gamma(p) = \int_0^\infty e^{-t} t^{p-1} dt$, $\quad (p > 0)$.

   Using the result $\Gamma(p+1) = p\Gamma(p)$, Compute each of the following:

   (a) $\dfrac{\Gamma 4.\Gamma 3.5}{\Gamma 5.5}$ (b) $\Gamma(-4.7)$

   (ii) The Beta function denoted by $\beta(p,q)$ is defined by $\quad \beta(p,q) = \int_0^1 x^{p-1}(1-x)^{q-1} dx$,

   where $p > 0$ and $q > 0$ are parameters.

   Evaluate each of the following:

   (a) $\displaystyle\int_0^\infty \frac{x^2 \, dx}{\left(1+x^4\right)^3}$.   (b) $\displaystyle\int_0^2 \frac{x^2 \, dx}{\sqrt{2-x}}$ (c) $\displaystyle\int_0^a y^4 \sqrt{a^2 - y^2} \, dy$.

2. Let $J_p(x)$ be the Bessel function of order p given by the expansion

$$J_p(x) = x^p \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m}}{2^{2m+p} . m! \Gamma(p+m+1)}.$$

(i) Find $J_0(x)$ and $J_1(x)$.

(ii) Show that $J_n(x)$ is an even function when n is even and an odd function when n is odd.

(iii) Show that $J_{\frac{3}{2}}(x) = \sqrt{\frac{2}{\pi x}} \left(\frac{\sin x}{x} - \cos x \right)$.

(iv) Evaluate $\int J_3(x) dx$.

(Hint: You may use the following recurrence relations, if necessary without proof.)

$$\frac{d}{dx} \{ x^{-p} J_p(x) \} = -x^{-p} J_{p+1}(x).$$

$$J'_p(x) = \frac{1}{2} \{ J_{p-1}(x) - J_{p+1}(x) \}$$

$$J_{p-1}(x) + J_{p+1}(x) = \frac{2p}{x} J_p(x)$$