



Duration: One Hour

Date:- 22.04.2017

Time:- 10:30a.m. - 11:30a.m.

ANSWER ALL QUESTIONS.

1. (a) State the domain and the range of the function $f(x, y) = \sqrt{y - x^2}$ and sketch the domain.

(b) Sketch the level curves of the function $f(x, y) = (x-1)^2 + (y+1)^2$.

(c) Find the value of the following limits, if they exist:

$$(i) \lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{x^2 + y^4}, \quad (ii) \lim_{(x,y) \rightarrow (0,0)} \frac{x^3}{x^2 + y^2}.$$

(d) Discuss the continuity of the following function at the origin:

$$f(x, y) = \begin{cases} \frac{x^3}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$$

2.(a)(i) If $w = x^3 + xy$ where $x = \cos t$ and $y = \sin t$ then compute dz/dt in two different methods.

(ii) If $z = x^2 - y^2$ and $x = \frac{1}{2}(u^2 - v^2)$ and $y = uv$, show that $u \frac{\partial z}{\partial v} - v \frac{\partial z}{\partial u} = 2 \left(x \frac{\partial z}{\partial y} - y \frac{\partial z}{\partial x} \right)$.

(b) Find the equation of the tangent plane to the surface $z^2 - 2x^2 - 2y^2 = 12$ at the point $(1, -1, 4)$.

(c) The temperature in degrees Celsius on the surface of a metal plate is $T(x, y) = 20 - 4x^2 - y^2$ where x and y are measured in centimeters. In what direction from the point $(2, -3)$ does the temperature increase most rapidly?
What is this rate of increase?

(d) Find all local maxima and minima of the function $f(x, y) = x^3 - 3x + xy^2$ and determine their nature.