00207

The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme
Final Examination 2015/2016
Applied Mathematics – Level 04

APU2142/ APE4142 – Newtonian Mechanics I

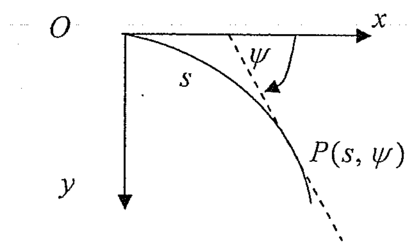
Duration :- Two Hours

Date :-11.01.2017 **Time:- 01.30p.m.-3.30 p.m.**

Answer Four Questions Only.

1. (i) With the usual notation, show that in intrinsic coordinates the velocity and acceleration components of a particle moving in a plane curve are given by $\underline{v} = \dot{s}\,\underline{t}$ and $\underline{a} = \ddot{s}\,\underline{t} + \dfrac{\dot{s}^2}{\rho}\,\underline{n}$ respectively.

(ii) The figure shows a curve C that forms the vertical cross-section of a smooth surface. A particle P moves in a vertical plane along the curve C, whose intrinsic equation is $s = 3a\tan\psi,\ 0 \le \psi < \pi/2$.



The coordinates $(s,\ \psi)$ of P are measured relative to a fixed point O and a fixed horizontal line Ox. The particle of unit mass is released from rest from the point $(a\sqrt{3},\ \pi/6)$ and slides down the surface along C.

(a) Write down the equations of motion when the particle is at $(s,\ \psi)$ and show that, while the particle remains in contact with the surface, the speed v of the particle is given by

$$v^2 = 6ga\left(\sec\psi - \frac{2}{\sqrt{3}}\right).$$

(b) Find the value of ψ when the particle leaves the surface and deduce that the distance travelled along the curve by the particle is $(\sqrt{39} - \sqrt{3})a$.

2. (i) With the usual notation, show that in plane polar coordinates, the velocity and acceleration components of a particle moving in a plane are given by $\underline{v} = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta$ and

$$\underline{a} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + \frac{1}{r}\frac{d(r^2\dot{\theta})}{dt}\underline{e}_\theta$$ respectively.

(ii) A Particle P of mass m moves along a plane curve with polar equation $r = a\sec^2(\theta/2)$ where a is a positive constant in such a way that u, the transverse component of the velocity remains constant. Assume that at $t = 0,\ \theta = 0$.

(a) Obtain an equation connecting θ and t and show that the magnitude of the horizontal radial component of velocity of P has magnitude $|u\tan(\theta/2)|$.

(b) Find the radial and transverse components of the acceleration of P.

(c) If the curve is fixed in a horizontal plane, find the resultant horizontal force acting on P.

3. (i) (a) With the usual notation show that the equation of a central orbit is given by

$$\frac{d^2u}{d\theta^2} + u = \frac{F}{h^2u^2} \quad \text{and} \quad \dot{\theta} = hu^2.$$

(b) A particle of mass m describes an ellipse with the centre of force at the centre of the ellipse. Find the force.

(ii) Let the masses of earth and moon be M_e and M_m respectively and the distance between their centres of mass be d. Also, let C_m and C_e be centres of the moon and earth rspectively. Show that the distance from C_e to the point along the line $C_e C_m$ at which the gravitational attractions due to these two bodies are equal and opposite is $\frac{kd}{1+k}$ where

$$k^2 = \frac{M_e}{M_m}.$$

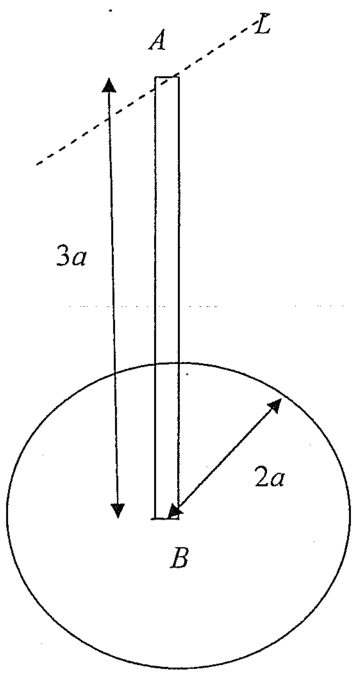
4. (i) Establish the formula $\underline{F}(t) = m(t)\frac{d\underline{v}}{dt} + \underline{u}\frac{dm}{dt}$ for the motion of a particle of varying mass $m(t)$ moving with velocity $\underline{v}$ under a force $\underline{F}(t)$, the matter being emitted at a rate $\frac{dm}{dt}$ with velocity $\underline{u}$ relative to the particle.

(ii) A particle falls from rest under gravity through a stationary cloud. The mass of the particle increases by accretion from the cloud at a rate which at any time is mkv, where m is the mass and v is the speed of the particle, and k is a constant. Show that, after the particle has fallen a distance x , v is given by

$$kv^2 = g(1 - e^{-2kx})$$

and find the distance the particle has fallen after time t.

5. A pendulum P is modelled as a uniform rod AB of length $3a$ and mass M, rigidly fixed to a uniform circular disc of mass M and radius $2a$. The end B of the rod is attached to the centre of the disc, and the rod lies in the plane of the disc, as shown in in the following figure. The pendulum is free to rotate in a vertical plane about a fixed smooth horizontal axis L, which passes through the end A and is perpendicular to the plane of the disc.



(a) Show that the moment of inertia of the body about L is $14Ma^2$.

(b) The pendulum makes small oscillations about L. Find the period of these small oscillations.

6. (a) Show that if there is no net external torque acting on a system, then the total angular momentum of the system is constant.

(b) A uniform square lamina of mass m and side length l, is free to rotate about a fixed smooth horizontal axis which coincides with a side of the lamina. The lamina is hanging in equilibrium when it is hit at its centre of mass by a particle of mass $3m$ moving with speed v in a direction perpendicular to the plane of the lamina. The particle then adheres to the lamina. Find, in terms of v and l, the angular speed of the lamina immediately after the impact. Hence show that the lamina will perform complete revolutions if $v^2 > \frac{104}{27}lg$.