The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme
Final Examination-2015/2016
APU 2144/APE4144- Applied Linear Algebra and Differential Equations
APPLIED MATHEMATICS-LEVEL 04

Duration: Two Hours.

Date: 22.01.2017	**Time: 09.30 a.m. - 11.30 a.m.**

Answer FOUR questions only.

1. (a) Showing your steps clearly, find the inverse of the matrix B where

$$B = \begin{pmatrix} 2 & 5 & 2 & 3 \\ 2 & 3 & 3 & 4 \\ 3 & 6 & 3 & 2 \\ 4 & 12 & 0 & 8 \end{pmatrix}$$

(b) Determine the non-singular matrices P and Q such that PAQ is in the normal form of A

where $A = \begin{pmatrix} 1 & -1 & 2 & -1 \\ 4 & 2 & -1 & 2 \\ 2 & 2 & -2 & 0 \end{pmatrix}$

What is the rank of A?

(c) Let $A = \begin{pmatrix} 1 & -a & 1 \\ b & 0 & 2b \\ 0 & a & 0 \end{pmatrix}$.

Prove that, if I refers to the identity matrix and $ab \neq 0$

(i) $A^{-1} = \dfrac{1}{ab}\left(A^2 - A - abI\right)$, and

(ii) $A^n - abA^{n-2} = A^2 - abI$, for every integer $n \geq 3$.

2. (a) Prove each of the following:

(i) A^{-1} exists if and only if 0 (zero) is not an eigen value of A.

(ii) If λ is an eigen value of a matrix A, and m is a positive integer, then A^m has an eigen value λ^m.

(b) Find the orthogonal transformation which transforms the quadratic form

$2x_1^2 + x_2^2 - 3x_3^2 - 8x_2x_3 - 4x_3x_1 + 12x_1x_2$, to a canonical form.

3. (a) Find the general solutions of each of the systems of simultaneous differential equations, given below. Here, the dots signify a standard notation involving the parameter t.

(i) $\dot{x}_1 = -x_1 + x_2 + 3e^{4t}$
$\dot{x}_2 = -12x_1 + 6x_2 + 8e^{2t}$

(ii) $\ddot{x}_1 = x_1 + 3x_2 - x_3$
$\ddot{x}_2 = 3x_1 + x_2 + x_3$
$\ddot{x}_3 = 4x_3$

(b) Find the general solution (complementary function and sinusoidal particular solution) of the following differential equation:

$\ddot{x} + 2\dot{x} + 5x = 2\cos 3t$.

Also find the solution for large positive values of t.

4. (a) Use a suitable change of variable to find the general solution of the differential equation

$$2(x-\mu)^2\frac{d^2y}{dx^2} + 11(x-\mu)\frac{dy}{dx} - 5y = 0, \; x > \mu,$$ where μ is a constant.

(b) Find, using the integrating factor method, the general solution of the partial differential equation

$$\frac{\partial u}{\partial x}+\frac{1}{x^2(1+y)}u=-2(1-y)\exp\left(\frac{1}{x(1+y)}\right) \quad ; (x\neq 0, y\neq -1).$$

(c) Suppose that u is a function of two variables x and t, satisfying the partial differential

equation $\frac{\partial^2 u}{\partial t^2}-c^2\frac{\partial^2 u}{\partial x^2}=0$, where c is a non-zero constant.

By transforming to new variables ζ and ϕ, where $\zeta=x-ct$ and $\phi=x+ct$, show that the equation can be simplified to $\frac{\partial^2 u}{\partial\zeta\partial\phi}=0$.

Hence write down the solution of the original partial differential equation.

5. (a) Find the equations of the characteristic curves for the partial differential equation

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+u=2$$

and define new variables that could be used to simplify the equation.

(b) Hence obtain the general solution $u(x, y)$ of the partial differential equation given in part (a).

(c) The partial differential equation $\frac{\partial u}{\partial x}-3\frac{\partial u}{\partial y}=u$ has the solution $u(x,y)=e^x f(y+3x)$.

If $u(x, y)=y$ on $x=0$, what is the solution in this case?

6. (a) Consider the partial differential equation

$$y^2\frac{\partial^2 u}{\partial x^2}-2xy\frac{\partial^2 u}{\partial x\partial y}+x^2\frac{\partial^2 u}{\partial y^2}-\frac{y^2}{x}\frac{\partial u}{\partial x}-\frac{x^2}{y}\frac{\partial u}{\partial y}=0, \quad (x\neq 0,\ y\neq 0).$$

Use the transformation $\zeta=x^2+y^2$ and $\phi=y$ to show that it can be reduced to the

standard form $\frac{\partial^2 u}{\partial\phi^2}-\frac{1}{\phi}\frac{\partial u}{\partial\phi}=0$

Use the method of characteristics to derive these expressions for ζ and ϕ.

(b) Solve each of the following partial differential equations:

(i) $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0.$

(ii) $2\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial x \partial y} - 6\frac{\partial^2 u}{\partial y^2} = 0.$