

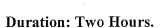
The Open University of Sri Lanka

B.Sc./B.Ed. Degree Programme

Final Examination-2015/2016

APU 2144/APE4144- Applied Linear Algebra and Differential Equations

APPLIED MATHEMATICS-LEVEL 04



Date: 22.01.2017

Time: 09.30 a.m. - 11.30 a.m.

Answer FOUR questions only.

1. (a) Showing your steps clearly, find the inverse of the matrix B where

$$B = \begin{pmatrix} 2 & 5 & 2 & 3 \\ 2 & 3 & 3 & 4 \\ 3 & 6 & 3 & 2 \\ 4 & 12 & 0 & 8 \end{pmatrix}$$

(b) Determine the non-singular matrices P and Q such that PAQ is in the normal form of A

where
$$A = \begin{pmatrix} 1 & -1 & 2 & -1 \\ 4 & 2 & -1 & 2 \\ 2 & 2 & -2 & 0 \end{pmatrix}$$

What is the rank of A?

(c) Let
$$A = \begin{pmatrix} 1 & -a & 1 \\ b & 0 & 2b \\ 0 & a & 0 \end{pmatrix}$$
.

Prove that, if I refers to the identity matrix and $ab \neq 0$

(i)
$$A^{-1} = \frac{1}{ab} (A^2 - A - abI)$$
, and

(ii)
$$A^n - abA^{n-2} = A^2 - abI$$
, for every integer $n \ge 3$.

- 2. (a) Prove each of the following:
 - (i) A^{-1} exists if and only if 0 (zero) is not an eigen value of A.
 - (ii) If λ is an eigen value of a matrix A, and m is a positive integer, then A^m has an eigen value λ^m .
 - (b) Find the orthogonal transformation which transforms the quadratic form

$$2x_1^2 + x_2^2 - 3x_3^2 - 8x_2x_3 - 4x_3x_1 + 12x_1x_2$$
, to a canonical form.

3. (a) Find the general solutions of each of the systems of simultaneous differential equations, given below. Here, the dots signify a standard notation involving the parameter *t*.

(i)
$$\dot{x}_1 = -x_1 + x_2 + 3e^{4t}$$

 $\dot{x}_2 = -12x_1 + 6x_2 + 8e^{2t}$

(ii)
$$\ddot{x}_1 = x_1 + 3x_2 - x_3$$

 $\ddot{x}_2 = 3x_1 + x_2 + x_3$
 $\ddot{x}_3 = 4x_3$

(b) Find the general solution (complementary function and sinusoidal particular solution) of the following differential equation:

$$\ddot{x} + 2\dot{x} + 5x = 2\cos 3t.$$

Also find the solution for large positive values of t.

4. (a) Use a suitable change of variable to find the general solution of the differential equation

$$2(x-\mu)^2 \frac{d^2y}{dx^2} + 11(x-\mu)\frac{dy}{dx} - 5y = 0$$
, $x > \mu$, where μ is a constant.

(b) Find, using the integrating factor method, the general solution of the partial differential equation

$$\frac{\partial u}{\partial x} + \frac{1}{x^2(1+y)}u = -2(1-y)\exp\left(\frac{1}{x(1+y)}\right) \quad ; (x \neq 0, y \neq -1).$$

(c) Suppose that u is a function of two variables x and t, satisfying the partial differential equation $\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0$, where c is a non-zero constant.

By transforming to new variables ζ and ϕ , where $\zeta = x - ct$ and $\phi = x + ct$, show that the equation can be simplified to $\frac{\partial^2 u}{\partial \zeta \partial \phi} = 0$.

Hence write down the solution of the original partial differential equation.

5. (a) Find the equations of the characteristic curves for the partial differential equation

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + u = 2$$

and define new variables that could be used to simplify the equation.

- (b) Hence obtain the general solution u(x, y) of the partial differential equation given in part (a).
- (c) The partial differential equation $\frac{\partial u}{\partial x} 3 \frac{\partial u}{\partial y} = u$ has the solution $u(x, y) = e^x f(y + 3x)$.

If u(x, y) = y on x = 0, what is the solution in this case?

6. (a) Consider the partial differential equation

$$y^{2} \frac{\partial^{2} u}{\partial x^{2}} - 2xy \frac{\partial^{2} u}{\partial x \partial y} + x^{2} \frac{\partial^{2} u}{\partial y^{2}} - \frac{y^{2}}{x} \frac{\partial u}{\partial x} - \frac{x^{2}}{y} \frac{\partial u}{\partial y} = 0, \quad (x \neq 0, \ y \neq 0).$$

Use the transformation $\zeta = x^2 + y^2$ and $\phi = y$ to show that it can be reduced to the

standard form
$$\frac{\partial^2 u}{\partial \phi^2} - \frac{1}{\phi} \frac{\partial u}{\partial \phi} = 0$$

Use the method of characteristics to derive these expressions for ζ and ϕ .

(b) Solve each of the following partial differential equations:

(i)
$$\frac{\partial^2 u}{\partial x^2} - 2 \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0.$$

(ii)
$$2\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial x \partial y} - 6\frac{\partial^2 u}{\partial y^2} = 0.$$