The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme
Final Examination 2015/2016
Applied Mathematics – Level 05

APU 3145/ APE5145 – Newtonian Mechanics II

Duration :- Two Hours

Date :-16.01.2017 **Time:-01.30 p.m. to 03.30 p.m.**

Answer Four Questions Only.

1. (a) In the usual notation, show that in spherical polar coordinates, the velocity and acceleration of a particle are given by $\dot{\underline{r}} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + r\sin\theta\dot{\phi}\underline{k}$ and

$$\ddot{\underline{r}} = \left(\ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2\sin^2\theta\right)\hat{r} + \left(\frac{1}{r}\frac{d}{dt}\left(r^2\dot{\theta}\right) - r\sin\theta\cos\theta\dot{\phi}\right) + \frac{1}{r\sin\theta}\frac{d}{dt}\left(r^2\sin^2\theta\dot{\phi}\right)\hat{\phi}$$

respectively.

(b) A particle of mass m moves inside a smooth sphere. The velocity of the particle at a point P is the same as that due to it falling freely from rest from the level of the centre to the point P. Show that the reaction of the surface will vary as the depth below the centre.

2. (a) Obtain, in the usual notation, the equation $\frac{\partial^2 \underline{r}}{\partial t^2} + 2\underline{\omega} \times \frac{\partial \underline{r}}{\partial t} = -g\underline{k}$ for the motion of a particle relative to the rotating earth.

(b) A particle of mass m is released from rest from a height h above the surface of the earth. Show that it reaches the earth at a point east of the vertical at a distance $\frac{2}{3}\omega h\cos\lambda\sqrt{\frac{2h}{g}}$ where λ is the attitude of the point of projection and ω is the angular speed of the earth about its polar axis.

03. In the usual notation, derive Lagrange's equations for a system with holonomic constraints $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q_j$, $j = 1, 2, \ldots, n$ where T is the kinetic energy of the system.

A light string passes over a fixed smooth pully. It carries a mass $6m$ at one end, the other end being attached to a smooth pulley of mass $3m$ over which passes a light string whose ends carry masses $2m$ and m. Assume that t6he portions of the strings not in contact with the pulleys are vertical and that the motion is confined to a vertical plane.

(a) Assuming the system starts from rest and neglecting moments of inertia of the pulleys write down the kinetic energy T and the generalized forces Q associated with each particle.

(b) Find the accelerations of the moveable pulley and the masses using only the result in the first part of this question.

4. (a) With the usual notation, show that the Lagrange's equations of motion for a conservative system with holonomic constraints are given by $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right)-\frac{\partial L}{\partial q_j}=0, \quad j=1,2\ldots,n.$

(You may assume the result in Question 3)

(b) A uniform circular hoop of mass M and radius a swings in a vertical plane about a point O of itself, which is freely hinged to a fixed support and a bead of mass m is free to slide along the hoop. Taking C as the centre of the hoop, B as the bead and the inclination of OC and CB to the downward vertical as θ, ϕ respectively, obtain two equations for the generalized coordinates θ and ϕ.

If the system is gently and slightly disturbed from its position of stable equilibrium, derive the equations

$$(2M+m)\ddot{\theta}+m\ddot{\phi}+(M+m)n^2\theta=0,$$

$$\ddot{\theta}+\ddot{\phi}+n^2\phi=0 \qquad \text{where } n^2=\frac{g}{a}.$$

5. (a) Derive Euler's equations of motion of a rigid body rotating about a fixed point.

(b) If a body moves under no forces about a point O and if H is the angular momentum about O and T the kinetic energy of the body then show that H and T are conserved.

(c) A rigid body moves about a point O under no forces, the principal moments of inertia of the body at O being $3A$, $5A$, $6A$. The body is initially rotated with an angular velocity $\underline{\omega}=(\omega_1, \omega_2, \omega_3)$ about the principal axes where $\omega_1=n$, $\omega_2=0$ and $\omega_3=n$. Show that at time t, $\omega_2=\frac{3n}{\sqrt{5}}\tanh\left(\frac{nt}{\sqrt{5}}\right)$.

6. (a) Define the Hamiltonian H of a holonomic system and derive in the usual notation, Hamilton's equations of motion, $\frac{\partial H}{\partial p_i}=\dot{q}_i$, $\frac{\partial H}{\partial q_i}=-\dot{p}_i$.

(b) The Hamiltonian of a dynamical system is given by $H=qp^2-qp+cp$ where c is a constant. Obtain Hamilton's equations of motion and hence find p and q at time t.