The Open University of Sri Lanka

B.Sc/B.Ed. DEGREE, CONTINUING EDUCATION PROGRAMME

FINAL EXAMINATION 2015/2016

Level 05 - Applied Mathematics

APU3147/APE5147– Statistical Inference

**Duration: - Two Hours.**

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**DATE: - 25-01-2017** **Time: - 9.30 a.m. to 11.30 a.m.**

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**Non programmable calculators are permitted. Statistical tables are provided.**

**Answer four questions only.**

1.

Suppose that the weight of a certain product, produced by ABC Company, follows a normal distribution. However, the mean and variance of the weight of a randomly selected product is unknown.

(i) What is the population of interest? Is the population finite? Justify your answer.

(ii) Derive moment estimators for the mean and the variance of the weight of a randomly selected product.

(iii) Weights of 16 randomly selected products in grams are given bellow. Using part (ii) estimate the mean and the variance of weight of a randomly selected product.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 49.07 | 50.01 | 49.06 | 49.74 | 49.79 | 49.82 | 49.62 | 47.99 |
| 49.83 | 48.98 | 49.22 | 50.87 | 50.11 | 50.14 | 50.42 | 45.99 |

(iv) Estimate the standard error of the estimated mean.

(v) Estimate the sample size required to estimate the mean weight of a randomly selected product with an error bound of 1 gram with 95% confidence.

(vi) Using a suitable statistical test, investigate he validity of the claim that "the variance of a randomly selected product is greater than 5 grams".

2.

In a production process of automotive crankshaft bearings, the production manager is interested in the proportion $\theta$ of nonconforming automotive crankshaft bearings produced. Suppose total production on a particular day is 10000. Random sample of 80 automotive crankshaft bearings (drawn with replacement) were tested on this particular day by the production manager. Suppose that 8 of the bearings had surface finish that is rougher than the specifications will allow.

(i) Construct a 95% confidence interval for $\theta$

(ii) Construct 95% confidence interval for the total number of nonconforming automotive crankshaft bearings in the production on that day. Hence comment on the claim that " total number of nonconforming automotive crankshaft bearings produced on that particular day is 110"

(iii) Using a suitable statistical test comment on the claim that "proportion $\theta$ of nonconforming automotive crankshaft bearings produced on that particular day is greater than 0.11"

(iv) Using Part (iii) test the validity of the claim that " total number of nonconforming automotive crankshaft bearings produced on that particular day exceeds 110"

3.

(a) Let $X_1, X_2, X_3, \ldots\ldots\ldots X_n$ be a random sample from a distribution with density given by $f(x;\theta)$. Let $\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \hat{\theta}_4$ are functions of $X_1, X_2, X_3, \ldots\ldots\ldots X_n$. Suppose $\hat{\theta}_1, \hat{\theta}_2\ \hat{\theta}_3, \hat{\theta}_4$ are unbiased estimators for parameter $\theta$ and $MSE(\ \hat{\theta}_3\ ) < MSE(\ \hat{\theta}_2\ )$. Prove or disprove each of the following statements:

(i) $Bias(\frac{\widehat{\theta_1}+\widehat{\theta_3}}{3} + \frac{\widehat{\theta_2}+3\widehat{\theta_4}}{12}) = 0$

(ii) $Var(\hat{\theta}_2) < Var(\hat{\theta}_3)$

(b) The diameter of steel rods manufactured on an extraction machine is being investigated. A random sample of size n = 13 were selected from the production of the machine. The sample mean and the sample variance are $\bar{x} = 8.73\ mm$, $s^2 = 0.35\ mm^2$ respectively. Assume that the diameter of steel rods manufactured by extraction machine follows a normal distribution.

(i) Construct a 95% confidence interval for the variance of diameter of a randomly selected steel rod manufactured by the machine and interpret your answer.

(ii) Dose the data provide evidence to justify the claim " mean diameter of a randomly selected steel rod is 10 mm". Using a suitable statistical test write your answer. Use 5% level of significance.

4.

An investigation was conducted in to the dust content in the flue gases of two types of solid – fuel boilers. Thirteen boilers of type $A$ and nine boilers of type $B$ were used under identical fuelling and extraction conditions. Over a similar period, the following quantities, in grams, of dust were deposited in similar traps inserted in each of the twenty- two flues.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Type A | 73.1 | 56.4 | 82.1 | 67.2 | 78.7 | 75.1 | 48.0 |
| | 53.3 | 55.5 | 61.5 | 60.6 | 55.2 | 63.1 | |
| Type B | 53.0 | 39.3 | 55.8 | 58.8 | 41.2 | 66.6 | 46.0 |
| | 56.4 | 58.9 | | | | | |

Assume that these samples come from normal populations. Sample means of dust contents of type A and type B are 63.83 grams and 52.89 grams respectively. Sample standard deviations of the dust contents of type A and type B are 10.63 grams and 9.00 grams respectively.

(i) Test for an equality of population variances. Use 1% level of significance. Clearly state the findings.

(ii) Test for an equality of population means. Use 1% level of significance. Clearly state the findings.

(iii) Do the dust contents in the flue gases of two types of solid – fuel boilers have Identical distribution? Justify your answer.

(iv) Construct 90% confidence interval for the differences of mean dust contents of type A and type B. comment on the results.

5.

(a) Briefly explain the following terms.

(i) Type I error and Type II error

(ii) Power of the test.

(b)

Let $X_1, X_2, X_3, \dots\dots\dots X_n$ be a random sample from a uniform distribution with density given by

$$f(x;\theta) = \frac{1}{1+\theta} \quad ; \; 0 \leq x \leq 1+\theta \; ; \quad \theta > 0$$

(i) Prove that he mean of the above distribution is $\frac{1+\theta}{2}$.

(ii) Derive a moment estimator for $\theta$. Is the moment estimator derived by you, an unbiased estimator for $\theta$? Prove your answer.

(iii) Derive Maximum likelihood estimators for $\theta$ and for mean of the above distribution.

(iv) A random sample drawn from the above distribution is given in the following table.

| | | | | |
|---|---|---|---|---|
| 0.49 | 1.38 | 1.95 | 1.28 | 1.80 |
| 0.21 | 0.05 | 0.48 | 0.49 | 1.40 |
| 0.59 | 0.23 | 0.26 | 1.65 | 1.43 |
| 1.56 | 0.16 | 0.82 | 0.37 | 0.80 |

[i] Estimate $\theta$ using moment estimator derived in part(ii).

[ii] Estimate $\theta$ and mean of the above distribution using maximum likelihood estimators derived in part(iii)

6.

(a) Suppose $\hat{\theta}$ is an estimator for parameter $\theta$. State whether the following statements are true or false. In each case justify your answer.

(i) $\hat{\theta}$ is an unbiased estimator for parameter $\theta$ implies that $\hat{\theta}$ is a consistent estimator for parameter $\theta$

(ii) $Var(\hat{\theta}) = \frac{\theta}{n}$ and $\hat{\theta}$ is an unbiased estimator for parameter $\theta$ implies that $\hat{\theta}$ is a consistent estimator for parameter $\theta$.

(b) Assignment marks and final examination marks of a particular subject for 15 students are given below.

| Student Name | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Assignment mark | 60 | 47 | 60 | 56 | 47 | 27 | 45 | 61 | 68 | 62 | 35 | 53 | 57 | 25 | 31 |
| Final Mark | 67 | 54 | 53 | 49 | 47 | 35 | 30 | 77 | 57 | 54 | 42 | 60 | 63 | 42 | 28 |

Using suitable statistical test, test the validity of the claim that "Expected assignment mark is greater than the expected final examination mark for a randomly selected student". Use 5% level of significance.