The Open University of Sri Lanka
Department of Mathematics and Computer Science
B.Sc/B.Ed. Degree Programme
Final Examination- 2015/2016
Applied Mathematics-Level 05

APU3150/APE5150/AMU3181/AME5181-Fluid Mechanics

Duration:-Two hours

Date:- 07.01.2017	**Time:- 09.30 a.m – 11.30 a.m**

ANSWER FOUR QUESTIONS ONLY

1. (a) Briefly describe the each of the following types of fluid motions:

   i. Steady and Un-steady flows

   ii. Uniform and Non-uniform flows

   iii. Compressible and Incompressible flows

   iv. Rotational and Irrotational flows.

   (b) Find the acceleration components at a point $(1,1,1)$ for the flow field with velocity components (u, v, w), where

   $u = 2x^2 + 3y;\ v = -2xy + 3y^2 + 3zy;\ w = -(3/2)z^2 + 2xz - 9y^2 z.$

   (c) The velocity potential function for a flow is given by $\varphi = (x^2 - y^2)$. Verify that the flow is incompressible and determine the stream function for the flow.

2. (a) A vacuum pump is used to drain out water from a basement at $20^{o}C$. The vapor pressure of water at this temperature is 2.34 *KPa*. The pump is capable of lifting water only up to a height of 10.5 m above the basement. Show that the atmospheric pressure is approximately 105*KPa*.

   (b) An open separation tank contains brine to a depth of 2*m* and a 3*m* layer of oil on top of the brine. An uniform sphere is floating at the brine-oil interface with 80% of its volume submerged in brine. Density of brine is 1030 kg/m^3 and the density of oil is 880kg/m^3. Show that the density of the sphere is nearly 1000kg/m^3.

3. (a) With the usual notation, derive the continuity equation in the form $\frac{D\rho}{Dt} + \rho\, div(\underline{\mathbf{q}}) = 0$, for any arbitrary control volume of a moving fluid irrespective of its shape and size.

(b) Hence deduce the continuity equation, for an incompressible fluid in steady motion, terms of Cartesian Coordinates.

(c) The velocity q in a certain fluid flow is given by $\underline{\mathbf{q}} = x^2 y\underline{\mathbf{i}} + y^2 z\underline{\mathbf{j}} - (2xyz + yz^2)\underline{\mathbf{k}}$.

Show that this represents possible motion of an incompressible fluid.

(d) In a fluid motion with velocity components (u, v, w), two of these are given as $v = 2y^2$ and $w = 2xyz$,. Determine the third component such that they satisfy the continuity equation.

4. (a) Given Euler's equation of motion in the form $\underline{\mathbf{F}} - \frac{1}{\rho} grad\ p = \frac{D\underline{\mathbf{q}}}{Dt}$ for a perfect fluid,

show that it can be written in the form $\underline{\mathbf{F}} - \frac{1}{\rho} grad\ p = \frac{\partial \underline{\mathbf{q}}}{\partial t} + grad\left(\frac{q^2}{2}\right) - \underline{\mathbf{q}} \times curl\underline{\mathbf{q}}$.

(b) Using the result in Part (a) above, derive Bernoulli's equation for irrotational motion of an inviscid homogeneous fluid of constant density.

(c) Consider a horizontal nozzle discharging into the atmosphere. The inlet has a bore area of 500 mm^2 and the exit has a bore area of 250 mm^2. Assuming there is no energy loss calculate the flow rate when the inlet pressure is 400 Pa.

5. A stream of liquid has velocity V at infinity in the negative x-direction, the sphere $r = 2$, being a rigid boundary. Moreover, the velocity potential of the flow is given by $\phi = V\left(r + \frac{4}{r^2}\right)\cos\theta$.

(a) Derive the components of velocity and hence obtain Stokes stream function.

$$\left(\text{Hint}: -\frac{\partial \phi}{\partial r} = q_r = -\frac{1}{r^2 \sin\theta}\frac{\partial \psi}{\partial \theta} \text{ and } -\frac{1}{r}\frac{\partial \phi}{\partial \theta} = q_\theta = \frac{1}{r\sin\theta}\frac{\partial \phi}{\partial r}\right).$$

(b) Find the equation of a streamline which is at a distance a from the axis, at infinity, and show that such a streamline meets the plane $\theta = \frac{\pi}{2}$, at a point where $r = b$ given by $\left(b^2 - \frac{8}{b}\right) = a^2$.

6. The complex potential of a fluid flow is given by $W(z) = U\left(z + \frac{4}{z}\right)$ where U is positive constant.

(a) Obtain the equation for the streamlines and velocity potential lines and represent them graphically.

(b) Find the complex velocity at any point and determine its value far from the origin.

(c) Find the stagnation points of the flow.

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