The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme
Final Examination – 2015/2016
Level 05-Applied Mathematics
AMU 3185/AME 5185 – EM Theory & Special Relativity

Duration :- Two Hours

Date :- 18.01.2017 **Time :- 1.30 p.m. – 3.30 p.m.**

Answer Four Questions Only.

01. An imaginary open surface S is in the form of a spherical cap $r=a,\ 0\le\theta\le\alpha,$ $0\le\omega\le 2\pi,$ where (r,θ,ω) denote spherical polar coordinates. Define the flux of a vector $\underline{E}$ through S.

    (a) Use your definition to calculate the electric flux through S if a uniform electric field $\underline{E}$ acts parallel to the axis of symmetry.
    (b) What is the flux if $\underline{E}$ acts perpendicular to the axis of symmetry?
    (c) Determine the flux through S if a point charge Q is placed at the centre $(r=0)$. Deduce the flux for the particular case $\alpha=\pi$.

02. Show that, if $\underline{A}$ satisfies the equations

    $$div\underline{A}=0, \qquad \nabla^2\underline{A}=\frac{1}{c^2}\ddot{\underline{A}}$$

    and $\underline{E}$ and $\underline{H}$ are defined by the relations

    $$\underline{E}=-\frac{1}{c}\dot{\underline{A}}, \qquad \underline{H}=curl\,\underline{A},$$

    then $\underline{E}$ and $\underline{H}$ satisfy the Maxwell's equations,

    $$curl\,\underline{H}-\frac{1}{c}\dot{\underline{E}}=0, \qquad div\,\underline{H}=0$$

    $$curl\,\underline{E}+\frac{1}{c}\dot{\underline{H}}=0, \qquad div\,\underline{E}=0$$

    for the electromagnetic field in vacuuo. (Dots denote partial differentiation with respect to time)

    Show that $\underline{A}=\underline{i}\,a\cos\frac{2\pi}{\lambda}(z-ct)+\underline{j}\,a\sin\frac{2\pi}{\lambda}(z-ct)$, where a and λ are constants, is a possible solution, for the Maxwell's equations given above.

03. (a) Explain briefly the following terms.

(i) Electrical potential (ii) Potential difference

(b) Show that the potential V_r at a point distance r from a point charge Q is given by

$V_r = \frac{1}{4\pi\varepsilon}\frac{Q}{r}$, where ε – permittivity.

(c) A charge $2q$ is uniformly distributed inside an insulating material in the form of a sphere of radius r.

Show that the potential at a distance $\frac{r}{2}$ from the centre is $\frac{11q}{16\pi\varepsilon_0 r}$.

04. Point charges $q, -q'$ and $-q'$ are placed at points $O(0,0), A(a,0)$ and $B(-a,0)$ respectively, where q, q' and a all positive.

(i) Given that $q > 2q'$ show that the extreme line of force ending on A is issued from O making an angle α with OA, where $\alpha = 2\sin^{-1}\sqrt{\frac{q'}{q}}$.

(ii) Given that $q < 2q'$ show that the extreme line of force ending on A is issued from O making an angle β with OA produced, where $\beta = 2\cos^{-1}\sqrt{\frac{q}{2q'}}$.

05.(a) Derive an expression for the magnetic field at any point on the line passing through the centre and perpendicular to the plane of a circular loop which carries a current I.

(b) Derive an expression for the magnetic field at a point on the axis of a solenoid of radius R and N turns/metre, which carries a current I.

06. Derive the Lorezntz transformation equations.

Verify that the above equations can be expressed in the form

$$x' = x\cosh\alpha - ct\sinh\alpha$$

$$y' = y$$

$$z' = z$$

$$ct' = ct\cosh\alpha - x\sinh\alpha \quad \text{where} \quad \tanh\alpha = v/c.$$

Deduce that

$$x' - ct' = (x - ct)e^{\alpha},$$

$$\text{and } x' + ct' = (x + ct)e^{\alpha}.$$