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THE OPEN UNIVERSITY OF SRI LANKA
B.Sc. /B.Ed. Degree Programme
APPLIED MATHEMATICS-LEVEL 05
AMU3186/AME5186- Quantum Mechanics
Final Examination 2015/2016

Duration: Two Hours.

Date: 24.01.2017 **Time: 1.30 p.m- 3.30 p.m**

Answer Four Questions Only.

1. (i) Define the commutator [A, B] of two operators $\hat{A}$ and $\hat{B}$.

   (ii) Let $\hat{A}$, $\hat{B}$, $\hat{C}$ and $\hat{D}$ are given operators, prove that

   (a) $\left[\hat{A}+\hat{B},\hat{C}\right]=\left[\hat{A},\hat{C}\right]+\left[\hat{B},\hat{C}\right]$,

   (b) $\left[\hat{A},\hat{B}+\hat{C}+\hat{D}\right]=\left[\hat{A},\hat{B}\right]+\left[\hat{A},\hat{C}\right]+\left[\hat{A},\hat{D}\right]$,

   (c) $\left[\hat{A}\hat{B},\hat{C}\right]=\hat{A}\left[\hat{B},\hat{C}\right]+\left[\hat{A},\hat{C}\right]\hat{B}$,

   (d) $\left[\hat{A},\hat{B}\right]=-\left[\hat{B},\hat{A}\right]$.

   (iii) Prove that

   (a) $\left[\hat{y},\hat{P}_y\right]=i\hbar$

   (b) $\left[\hat{P}_{y,}\hat{y}^n\right]=-in\hbar y^{n-1}$

   where p has a standard meaning.

2. Consider a particle with normalized wave function

   $\Psi(x)=A\sin\left[\frac{\pi x}{2a}\right]$; $0\le x\le a$ where a is a constant.

   (i) Determine the normalization constant, A.

   (ii) Calculate the mean values of x, x^2 and $\hat{P}_x$ with respect to $\Psi(x)$.

3. An X-ray photon of wave length $\lambda=10^{-10}$ m is incident on a stationary electron, where

   $\lambda_c=\frac{h}{mc}$ is the Compton wave length and m is the mass of the electron.

(i) Show that for Compton scattering,

$\delta\lambda = \lambda' - \lambda = 2\lambda_c \sin^2\frac{\theta}{2}$, where λ is the wave length of the incident X –ray and λ' is the wave length of the X-ray scattered through an angle θ.

(ii) Calculate the Compton shift.

(iii) Calculate the kinetic energy of the recoiling electron, if $m_e = 9.108\times10^{-31}kg$, $c = 3\times10^8 ms^{-1}$, $h = 6.625\times10^{-34} Js$ and $\theta = 30^0$.

4. (i) If $\hat{A}$ is an operator corresponding to a quantum observable and $\langle\hat{A}\rangle$ is the corresponding expectation value,

Show that $\frac{d\langle\hat{A}\rangle}{dt} = \langle\frac{\partial\hat{A}}{\partial t}\rangle + \frac{1}{i\hbar}\langle[\hat{A},\hat{H}]\rangle$.

(ii) A particle of mass m and energy E moves in the positive x direction through a square hill.

Potential is defined by

$$V = \begin{cases} 0 & ;\ x < 0 \\ V_0 > 0 & ;\ 0 < x < a \\ 0 & ;\ a < x \end{cases}$$

Find the wave function $u(x)$ for each region given above, for the cases $E < V_0$ and $E > V_0$.

5. Consider a particle with normalized function given by

$$\psi(x,t) = \begin{cases} A(a-x)e^{-i\omega t} & ;0 < x < a \\ 0 & ;\text{elsewhere} \end{cases}$$

where a and ω are positive constants.

(i) Determine the normalization constant A.
(ii) Find the expectation values for the position and the square of the position.
(iii) Calculate (Δx).

6. The angular momentum of a particle is defined as a vector $\underline{L}$, such that $\underline{L} = \underline{r} \times \underline{p}$, where $\underline{p}$ is the momentum and $\underline{r}$ is the position vector of the particle with respect to a fixed origin O.

(i) Write down the Cartesian components $\hat{L}_x, \hat{L}_y, \hat{L}_z$ of the angular momentum operator.

(ii) Hence obtain the angular momentum operator in spherical polar coordinates (r,θ,ϕ).

You may use

$$\underline{\hat{\theta}} = Cos\ \theta .\ Cos\ \phi\ \underline{i} + Cos\ \theta\ Sin\ \phi \underline{j} - Sin\ \theta \underline{k} \text{ and } \underline{\hat{\phi}} = -Sin\ \phi\ \underline{i} + Cos\ \phi \underline{j}.$$

(iii)Prove the relation $\left[\hat{L}_x, \hat{L}_y\right] = i\hbar\hat{L}_z$. You may use $\left[\hat{P}_z, \hat{z}\right] = -i\hbar$.