## THE OPEN UNIVERSITY OF SRI LANKA

B.Sc. /B.Ed. Degree Programme

APPLIED MATHEMATICS-LEVEL 05

APU3141- Linear Programming

Final Examination 2016/2017

**Duration: Two Hours** 

Date: 06.08.2017 Time: 01.00 p.m- 03.00 p.m

## Answer four questions only

- (1) An aeroplane can carry a maximum of 200 passengers. A profit of Rs 1000 is made on each business class ticket and a profit of Rs 600 is made on each economy class ticket. The airline reserves at least 20 seats for business class. However, at least 4 times as many passengers prefer to travel by economy class than by the business class.
  - (i) Formulate the above as a Linear Programming model to maximize the total profit.
  - (ii) Using the graphical method, solve the model in part(i) to determine how many tickets of each type must be sold in order to maximize the profit for the airline.
  - (iii) Find the maximum profit.
- (2) a) Briefly explain the following terms in linear programming:
  - i) Feasible solution
  - ii) Objective function
  - iii) Unbounded solution
  - iv) Optimal solution.
  - b) The final simplex tableau for a maximization linear programming problem with two constraints is given below. Here,  $X_1$ ,  $X_2$  represent decision variables,  $s_1$ ,  $s_2$  represent slack variables of the two constraints and Z represent the objective function:

Basis	$X_{I}$	$X_2$	S <sub>I</sub>	S2	Solution
$X_{I}$	1	0	0.75	-0.01	7.5
$X_2$	0	1	-0.50	0.01	5
Z	. 0	0	5	0.10	550

(i) Is the solution feasible? Justify.

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(i) Is the solution feasible? Justify.

Turn over

- (ii) Is the solution optimal? Justify.
- (iii) What are the values of  $X_1$  and  $X_2$ , according to this table? What is the value of Z?
- (3) A large manufacturing company is closing three of its existing plants and intends to transfer some of its more skilled employees to three plants that will remain open. The number of employees available for transfer from each closing plant is as follows:

Closing plant	Transferable Employees		
1	60		
2	105		
3	70		

The above number of employees can be accommodated at the three plants remaining open according to the requirement given below:

Open Plants	<b>Employees Demanded</b>		
A	45		
В	90		
C	35		

Each transferred employee will increase product output per day at each plant as shown in the following table. The company wants to transfer employees so as to ensure the maximum increase in product output.

From	То				
	A	В	C		
1	5	8	6		
2	10	9	12		
3	7	6	8		

- (i) Formulate a linear programming model to determine the maximum profit.
- (ii) Find the initial solution using Vogel's Approximation method in order to maximize the profit.
- (iii) Find the optimal solution.

Contd.

(4) A Head of a department at a University has five instructors to be assigned to five different courses. All the instructors have taught the courses in the past and have been evaluated by the students. The rating for each instructor for each course is given in the following table (a perfect score is 100). The Head wants to know the optimal assignment of instructors to courses that will maximize the overall average evaluation.

Formulate this problem as an Assignment problem and hence, solve it:

Instructor	Course					
	A	В	C	D	E	
1	80	75	90	85	88	
2	95	90	90	97	92	
3	85	95	88	91	95	
4	93	91	80	84	90	
5	91	92	93	88	89	

(5) Soft drink manufacturing company produces an orange flavored soft drink by combining orange soda and orange juice. Each ounce of orange soda contains 0.5 oz of sugar and 1 mg of vitamin C. Each ounce of orange juice contains 0.25 oz of sugar and 3 mg of vitamin C. It costs Rs.2 to produce an ounce of orange soda and Rs.3 to produce an ounce of orange juice. Marketing Department has decided that each 10 oz bottle of that soft drink must contain at least 20 mg of vitamin C and at most 4 oz of sugar.

Use a Linear Programming model to determine how manufacturing company can meet Marketing Department's requirements at the minimum cost.

(6) Consider the following linear programming problem:

Maximize 
$$z = 3y_1 + 6y_2 + 3y_3$$
,  
Subject to  $3y_1 + 4y_2 + y_3 \le 2$ ,  
 $y_1 + 3y_2 + 2y_3 \le 1$ ,  
 $y_1, y_2, y_3 \ge 0$ .

- (i) Write the dual linear programme for the above primal problem.
- (ii) Solve the dual linear programme given in part (i) by using the dual simplex method. Hence, write the optimal solution to the primal problem.

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