00124

The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme
Final Examination-2016/2017
APU3143/APE5143-Mathematical Methods
Applied Mathematics -Level 05

Duration: Two Hours.

Date: 11.08.2017	Time: 09.30 a.m.- 11.30 a.m.

Answer FOUR questions only.

1. The Laplace transform of a function $f(t)$, denoted by $L[f(t)]$ is defined as

$L[f(t)] = F(s) = \int_0^{\infty} f(t)e^{-st}\,dt$ and the inverse transform as $L^{-1}\{F(s)\} = f(t)$.

(i) Show that $L^{-1}\left\{\cot^{-1}\left(\frac{s+a}{b}\right)\right\} = \frac{e^{-at}}{t}\sin bt$.

(ii) Find the inverse Laplace transforms of each of the following functions:

(a) $F(s) = \dfrac{2-5s}{(s-6)(s^2+11)}$.

(b) $F(s) = \dfrac{1}{(s^2+a^2)^2}$.

(c) $F(s) = \dfrac{s+1}{s^2(s^2+1)}$.

2. (i) Use the convolution theorem to find the inverse Laplace transform of $\dfrac{a}{s^2(s^2+a^2)^2}$.

(ii) Solve each of the following boundary value problems using the Laplace transform method:

(a) $\dfrac{d^2y}{dx^2} - 4y = e^{-3x}\sin 2x$, subject to the boundary conditions $y(0) = y'(0) = 0$.

(b) $\dfrac{d^2y}{dx^2} - 3\dfrac{dy}{dx} + 2y = x^2 + 5$, subject to the boundary conditions $y(0) = 1,\ y'(0) = 0$.

3. (i) Find the characteristic values and characteristic functions of the following Sturm-Liouville problem:

$$\frac{d^2y}{dx^2} + \lambda y = 0, \quad y(0) - y'(0) = 0, \quad y(\pi) - y'(\pi) = 0.$$

(ii) Check the orthogonality of the above characteristic functions with respect to the weight function 1 on the interval $0 \le x < \pi$.

4. (i) Consider the function $f(x)$ defined for $-L \le x \le L$ by

$$f(x) = \begin{cases} 0, & -L \le x < 0, \\ L, & 0 \le x \le L. \end{cases}$$

Find the trigonometric Fourier series of $f(x)$ in $-L \le x \le L$.

(ii) Consider the function $f(x)$ defined by $f(x) = 2L \quad ; 0 \le x \le L$

Find the Fourier Sine series and the Fourier Cosine series of f in $0 \le x \le L$.

5.(i) The Gamma function denoted by $\Gamma(p)$ corresponding to the parameter p is defined by the improper integral $\Gamma(p) = \int_0^\infty e^{-t} t^{p-1} dt, \quad (p > 0)$.

(a) Compute $\dfrac{\Gamma(4)\Gamma(3.5)}{\Gamma(5.5)}$.

(b) Prove that $\int_0^\infty e^{-ax} . x^{n-1} dx = \dfrac{\Gamma(n)}{a^n}$ where a and n are positive.

(ii) The Beta function denoted by $\beta(p,q)$ is defined by

$$\beta(p,q) = \int_0^1 x^{p-1}(1-x)^{q-1} dx,$$

where $p > 0$ and $q > 0$ are parameters.

Evaluate each of the following intrigals using Beta function:

(a) $\int_0^2 \frac{x^2 dx}{\sqrt{2-x}}$.   (b) $\int_0^{\frac{\pi}{2}} \cos^6 \theta d\theta$.

6. Let $J_p(x)$ be the Bessel function of order p given by the expansion

$$J_p(x) = x^p \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m}}{2^{2m+p}.m!\Gamma(p+m+1)}$$

Prove each of the following:

(i) $\frac{d}{dx}\{xJ_n.J_{n+1}\} = x\left[J_n^2 - J_{n+1}^2\right]$

(ii) $\frac{d}{dx}\left[J_n^2 + J_{n+1}^2\right] = 2\left[\frac{n}{x}J_n^2 - \frac{n+1}{x}J_{n+1}^2\right]$.

(iii) $J_n''' = \frac{1}{8}\left[J_{n-3} - 3J_{n-1} + 3J_{n+1} - J_{n+3}\right]$. ; where ''' denotes the standard notation.