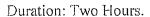
The Open University of Sri Lanka

B.Sc./B.Ed. Degree Programme

Final Examination-2016/2017

APU3143/APE5143-Mathematical Methods

Applied Mathematics -Level 05



Date: 11.08.2017

Time: 09.30 a.m.- 11.30 a.m.

Answer FOUR questions only.

1. The Laplace transform of a function f(t), denoted by L[f(t)] is defined as

$$L[f(t)] = F(s) = \int_{0}^{\infty} f(t)e^{-st} dt \text{ and the inverse transform as } L^{-1}\{F(s)\} = f(t).$$

(i) Show that 
$$L^{-1} \left\{ \cot^{-1} \left( \frac{s+a}{b} \right) \right\} = \frac{e^{-at}}{t} \sin bt$$
.

(ii) Find the inverse Laplace transforms of each of the following functions:

(a) 
$$F(s) = \frac{2-5s}{(s-6)(s^2+11)}$$
.

(b) 
$$F(s) = \frac{1}{(s^2 + a^2)^2}$$
.

(c) 
$$F(s) = \frac{s+1}{s^2(s^2+1)}$$
.

- 2. (i) Use the convolution theorem to find the inverse Laplace transform of  $\frac{a}{s^2(s^2+a^2)^2}$ .
  - (ii) Solve each of the following boundary value problems using the Laplace transform method:

(a) 
$$\frac{d^2y}{dx^2} - 4y = e^{-3x} \sin 2x$$
, subject to the boundary conditions  $y(0) = y'(0) = 0$ .

(b) 
$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = x^2 + 5$$
, subject to the boundary conditions  $y(0) = 1$ ,  $y'(0) = 0$ .

3. (i) Find the characteristic values and characteristic functions of the following Sturm-

Liouville problem:

$$\frac{d^2y}{dx^2} + \lambda y = 0, \quad y(0) - y'(0) = 0, \quad y(\pi) - y'(\pi) = 0.$$

- (ii) Check the orthogonality of the above characteristic functions with respect to the weight function 1 on the interval  $0 \le x < \pi$ .
- 4. (i) Consider the function f(x) defined for  $-L \le x \le L$  by

$$f(x) = \begin{cases} 0, & -L \le x < 0, \\ L, & 0 \le x \le L. \end{cases}$$

Find the trigonometric Fourier series of f(x) in  $-L \le x \le L$ .

(ii) Consider the function f(x) defined by f(x) = 2L;  $0 \le x \le L$ 

Find the Fourier Sine series and the Fourier Cosine series of f in  $0 \le x \le L$ .

5.(i) The Gamma function denoted by  $\Gamma(p)$  corresponding to the parameter p is defined by the

improper integral 
$$\Gamma(p) = \int_0^\infty e^{-t} t^{p-1} dt$$
,  $(p > 0)$ .

(a) Compute 
$$\frac{\Gamma(4)\Gamma(3.5)}{\Gamma(5.5)}$$
.

- (b) Prove that  $\int_0^\infty e^{-ax}.x^{n-1}dx = \frac{\Gamma(n)}{a^n}$  where a and n are positive.
- (ii) The Beta function denoted by  $\beta(p,q)$  is defined by

$$\beta(p,q) = \int_{0}^{1} x^{p-1} (1-x)^{q-1} dx,$$

where p > 0 and q > 0 are parameters.

Evaluate each of the following intrigals using Beta function:

(a) 
$$\int_0^2 \frac{x^2 dx}{\sqrt{2-x}}$$
. (b)  $\int_0^{\frac{\pi}{2}} \cos^6 \theta d\theta$ .

6. Let  $J_p(x)$  be the Bessel function of order p given by the expansion

$$J_{p}(x) = x^{p} \sum_{m=0}^{\infty} \frac{(-1)^{m} x^{2m}}{2^{2m+p} \cdot m! \Gamma(p+m+1)}$$

Prove each of the following:

(i) 
$$\frac{d}{dx} \{ x J_n J_{n+1} \} = x \left[ J_n^2 - J_{n+1}^2 \right]$$

(ii) 
$$\frac{d}{dx} \left[ J_n^2 + J_{n+1}^2 \right] = 2 \left[ \frac{n}{x} J_n^2 - \frac{n+1}{x} J_{n+1}^2 \right].$$

(iii) 
$$J_n''' = \frac{1}{8} [J_{n-3} - 3J_{n-1} + 3J_{n+1} - J_{n+3}]$$
.; where "denotes the standard notation.