The Open University of Sri Lanka
B.Sc. / B.Ed. Degree Programme – Level 05
Final Examination -2016/2017
Applied Mathematics
APU3240/APE5240 — Numerical Methods

064

Duration: Three Hours

Date: 23. 07. 2017 Time: 01.00 p.m. – 04.00 p.m.

Answer Five Questions Only.

1. (a) Derive method of False Position for solving the equation $f(x)=0$.

   (b) Find the root of the equation $xe^x - 2 = 0$, lying in the interval [1, 2], correct upto two decimal places using method of False Position.

   (c) Find the maximum number of iterations required to find the root of the equation $x^3 + x^2 - 1 = 0$, lying in the interval [0, 1], correct upto three decimal places.

2. (a) Prove that

   (i) $E = \Delta + 1$, (ii) $E = (1-\nabla)^{-1}$, (iii) $\delta = E^{1/2} - E^{-1/2}$,

   (iv) $\Delta = \nabla E = \delta E^{1/2}$ (v) $\mu = \sqrt{1 + \frac{\delta^2}{4}}$

   where Δ, ∇, δ, E and μ are the forward difference, the backward difference, the central difference, the shift and the averaging operators respectively.

   (b) Derive Gregory- Newton backward interpolation formula.

   (c) The following data are taken from the steam table.

Temperature $^\circ C$	140	150	160	170	180
Pressure N/cm^2	3.685	4.854	6.302	8.076	10.225

   Find the pressure at $t = 175^\circ C$ using Gregory- Newton backward interpolation formula.

3. (a) (i)Derive Newton's general interpolation formula with divided differences.

(ii) Find a polynomial $f(x)$, that passes through the points (1, 1), (2, 5), (7, 5) and (8, 4).

(b) (i) Derive Lagrange's interpolation formula.

(ii) Applying Lagrange's formula, obtain a polynomial $f(x)$ that passes through the points (5, 12), (6, 13), (9, 14) and (11, 16).

4. (a) Derive the Simpson's One –Third Rule.

(b) If the interval $[a, b]$ is divided into $2n$ sub intervals and corresponding ordinates are denoted by $y_0, y_2 \ldots\ldots y_{2n}$ then show that the error in Simpson's One –Third rule is given by $|E| < \frac{(b-a)h^4}{180}M$, where M is the numerically greater value of $y_0^{iv}, y_2^{iv} \ldots\ldots y_{2n-2}^{iv}$.

(c) Applying Simpson's One third rule for following data,

x	0	1/6	1/3	1/2	2/3	5/6	1
$\frac{1}{1+x^2}$	1	0.9730	0.9000	0.8000	0.6923	0.5902	0.5000

evaluate the integral $\int_0^1 \frac{1}{1+x^2}dx$. Hence find an approximate value for π.

5. (a) Applying Taylor series method of fourth order for the differential equation

$\frac{dy}{dx} = 1 - 2xy$ subject to the initial condition $y(0) = 0$, evaluate $y(0.2)$ and $y(0.4)$.

(b) Applying Taylor series method of fourth order for the differential equation $y'' - xy'^2 + y^2 = 0$ subject to the initial conditions $y(0) = 1$ and $y'(0) = 0$, evaluate $y(0.1)$, $y(0.2)$.

6. (a) (i) Derive formula for Picard's method to solve $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$.

(ii) Using Picard's method, find the first-three successive approximations to solve $\frac{dy}{dx} = 2x - y^2$ with the initial condition $y(0) = 0$.

(b) Applying Modified Euler method for $\frac{dy}{dx} = x\sqrt{1+y^2}$ subject to the initial condition $y(1) = 0$, evaluate $y(1.4)$ and $y(1.8)$.

7. (a) State fourth order Runge-Kutta algorithm to solve $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$.

(b) Solve $\frac{dy}{dx} = y - x^2$ with the initial condition $y(0) = 1$ using Runge-Kutta method of fourth order. Evaluate the value of y, when $x = 0.2$ and $x = 0.4$.

(c) Solve the system of differential equations $\frac{dy}{dx} = -xz$ and $\frac{dz}{dx} = y^2$ with the initial conditions $y(0) = 1$, $z(0) = 1$ using Runge-Kutta method of fourth order and evaluate $y(0.2)$ and $z(0.2)$.

8. (a) State Milne's Predictor – Corrector Method to solve $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$.

(b) Applying Taylor series method for $\frac{dy}{dx} = 2e^x - y$ with the initial condition $y(0) = 2$

show that $y(0.1) = 2.010$, $y(0.2) = 2.040$ and $y(0.3)=2.090$. Hence find $y(0.4)$ correct upto three decimal places by using Milne's Predictor – Corrector Method.