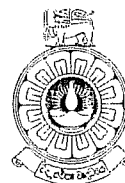


The Open University of Sri Lanka  
 Bachelor of Science Degree Programme – Level 05  
 Department of Computer Science  
 Final Examination 2016/2017  
 CPU3140 – Mathematics for Computing  
 Duration: Two hours only



Date: 22.07.2017

Time: 9.30 am – 11.30 am

Answer **FOUR** Questions only.

- (01) (i) Describe the principle of Mathematical Induction for a statement  $P(n)$  where  $n \in \mathbb{N}$
- (ii) Prove the following equation using the above principal for all  $n \geq 1$   

$$1+4+7+\dots+(3n-2) = \frac{n(3n-1)}{2}$$
- (iii) Use Mathematical Induction to verify that for all  $n \in \mathbb{N}$ , that  $(6^n - 1)$  is divisible by 5
- (02) (i) Define the following terms regarding matrices
- (a) A square matrix
  - (b) Identity matrix of any order
  - (c) Symmetric matrix
  - (d) Singular matrix
- (ii) Let  $A = \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix}$  and  $I$  be  $2 \times 2$  identity matrix.
- (a) Prove that  $A^2 = 7A + 2I$
  - (b) Hence, show that  $A^{-1} = \frac{1}{2}(A - 7I)$
- (iii) Let  $B = \begin{bmatrix} 5 & 2 & 3 \\ 4 & 7 & 1 \\ 8 & 5 & 9 \end{bmatrix}$
- Is "B" a symmetric matrix? Justify your answer.
- (iv) Let  $B_1 = \begin{bmatrix} 3 & 2 \\ 1 & 3 \end{bmatrix}$  Find the determinant of  $B_1$  and hence Find  $\det(B_2)$ .
- Where  $B_2 = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 3 & 2 \\ 0 & 1 & 3 \end{bmatrix}$

(03) (a) Give the definition of a Recurrence Relation.

(b) Clarify the following recurrence relations.

(i)  $a_n = a_{n-1} + 2a_{n-2} + 3a_{n-3}$

(ii)  $P_n = (1.12) P_{n-1}$

(iii)  $a_n = a_{n-1} + a_{n-2}^2$

(iv)  $f_n = f_{n-1} + f_{n-2} - f_{n-3} + 9$

(v)  $B_n = n^2 B_{n-1}$

(c) Solve the recurrence relation given below.

$$a_n = a_{n-1} + 2a_{n-2} \text{ with } a_0 = 2 \text{ and } a_1 = 7$$

(04) (i) Define the following terms regarding sets.

(a) Sets and Subsets

(b) Power set

(c) Null set

(ii) List the elements of the following sets where  $P = \{ 1, 2, 3, \dots \}$

(a)  $A = \{ x : x \in P, 3 < x < 12 \}$

(b)  $B = \{ x : x \in P, x \text{ is even and } x < 15 \}$

(c)  $C = \{ x : x \in P, 4 + x = 3 \}$

(d)  $D = \{ x : x \in P, x \text{ is a multiple of } 5 \}$

(iii) Using the laws of Algebra of sets, prove the following identities.

(a)  $(A \cap B) \cup (A \cap B^c) = A$

(b)  $A \cap (B \setminus C) = (A \cap B) \setminus (A \cap C)$

(iv) A small college of 140 students requires its students to take at least one mathematics course and at least one science course.

Use a Venn diagram or any other method in set theory to find the number of students who had completed:

(a) Exactly one of the two requirements.

(b) At least one of the requirements.

(c) Neither requirement.

(05) (i) Give the definition of a function and write a brief statement about the terms :

(a) Domain

(b) Co-domain

(c) The range of a function

(ii) Find the domain and the range of the following functions.

(a)  $f(x) = 2-x$ ,  $x \in \mathbb{N}$

(b)  $f(x) = x$ ,  $x \in \mathbb{R}$

(iii) Let the function  $f: \mathbb{R} \rightarrow \mathbb{R}$  be defined as follows:

$$f(x) = \begin{cases} 3x-1 & \text{if } x > 3 \\ x^2 - 2 & \text{if } -2 \leq x < 3 \\ 2x+3 & \text{if } x < -2 \end{cases}$$

Find the values of

(a)  $f(2)$ , (b)  $f(4)$  and (c)  $f(-3)$

(iv) The two functions  $f(x)$  and  $g(x)$  are

$$f(x) = x^2 + 3x + 1 \quad \text{and}$$

$$g(x) = 2x - 3 \quad \text{and } x \in \mathbb{R}$$

Find the composition functions of (a)  $f \circ g(x)$  and (b)  $g \circ f(x)$

(vi) What can you say about the two composition functions in part (iv) above. Justify your answer.

(06) (i) If  $p$  and  $q$  are two propositions, give the truth tables for Conjunction, Disjunction, Conditional implication and Bi-conditional.

(ii) Using part (i) construct the truth tables and verify whether each of the following is a tautology, a contradiction or a contingency. Justify your answers.

(a)  $(p \wedge q) \wedge \sim(p \vee q)$

(b)  $(p \vee \sim q) \rightarrow (p \wedge q)$

(c)  $(p \rightarrow q) \leftrightarrow (p \wedge \sim q)$

\*\*\*All Rights Reserved\*\*\*