00125

The Open University of Sri Lanka
Bachelor of Science Degree Programme – Level 05
Department of Computer Science
Final Examination 2016/2017
CPU3140 – Mathematics for Computing
Duration: Two hours only

Date: 22.07.2017 Time: 9.30 am – 11.30 am

Answer **FOUR** Questions only.

(01) (i) Describe the principle of Mathematical Induction for a statement P(n) where $n \in \mathbb{N}$

(ii) Prove the following equation using the above principal for all $n \geq 1$

$$1+4+7+\ldots\ldots\ldots+(3n-2) = \frac{n(3n-1)}{2}$$

(iii) Use Mathematical Induction to verify that for all $n \in \mathbb{N}$, that $(6^n - 1)$ is divisible by 5

(02) (i) Define the following terms regarding matrices

(a) A square matrix
(b) Identity matrix of any order
(c) Symmetric matrix
(d) Singular matrix

(ii) Let $A= \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix}$ and I be 2×2 identity matrix.

(a) Prove that $A^2 = 7A + 2I$
(b) Hence , show that $A^{-1} = \frac{1}{2}(A - 7I)$

(iii) Let $B = \begin{bmatrix} 5 & 2 & 3 \\ 4 & 7 & 1 \\ 8 & 5 & 9 \end{bmatrix}$

Is "B" a symmetric matrix? Justify your answer.

(iv) Let $B_1 = \begin{bmatrix} 3 & 2 \\ 1 & 3 \end{bmatrix}$ Find the determinant of B_1 and hence Find det(B_2).

Where $B_2 = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 3 & 2 \\ 0 & 1 & 3 \end{bmatrix}$

(03) (a) Give the definition of a Recurrence Relation.

(b) Clarify the following recurrence relations.

(i) $a_n = a_{n-1} + 2a_{n-2} + 3a_{n-3}$

(ii) $P_n = (1.12)\, P_{n-1}$

(iii) $a_n = a_{n-1} + a^2_{n-2}$

(iv) $f_n = f_{n-1} + f_{n-2} - f_{n-3} + 9$

(v) $B_n = n^2 B_{n-1}$

(c) Solve the recurrence relation given below.

$a_n = a_{n-1} + 2a_{n-2}$ with $a_0 = 2$ and $a_1 = 7$

(04) (i) Define the following terms regarding sets.

(a) Sets and Subsets

(b) Power set

(c) Null set

(ii) List the elements of the following sets where $P = \{ 1, 2, 3, \ldots \}$

(a) $A = \{ x : x \in P,\ 3 < x < 12 \}$

(b) $B = \{ x : x \in P,\ x \text{ is even and } x < 15 \}$

(c) $C = \{ x : x \in P,\ 4 + x = 3 \}$

(d) $D = \{ x : x \in P,\ x \text{ is a multiple of } 5 \}$

(iii) Using the laws of Algebra of sets, prove the following identities.

(a) $(A \cap B) \cup (A \cap B^C) = A$

(b) $A \cap (B \setminus C) = (A \cap B) \setminus (A \cap C)$

(iv) A small college of 140 students requires its students to take at least one mathematics course and at least one science course.
Use a Venn diagram or any other method in set theory to find the number of students who had completed:

(a) Exactly one of the two requirements.

(b) At least one of the requirements.

(c) Neither requirement.

(05) (i) Give the definition of a function and write a brief statement about the terms :

(a) Domain

(b) Co-domain

(c) The range of a function

(ii) Find the domain and the range of the following functions.

(a) $f(x) = 2-x$, $x \in \mathbb{N}$

(b) $f(x) = x$, $x \in \mathbb{R}$

(iii) Let the function $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined as follows:

$$f(x) = \begin{cases} 3x-1 & \text{if } x>3 \\ x^2 - 2 & \text{if } -2 \leq x<3 \\ 2x+3 & \text{if } x<-2 \end{cases}$$

Find the values of

(a) $f(2)$, (b) $f(4)$ and (c) $f(-3)$

(iv) The two functions $f(x)$ and $g(x)$ are

$f(x) = x^2 + 3x + 1$ and

$g(x) = 2x - 3$ and $x \in \mathbb{R}$

Find the composition functions of (a) $fog(x)$ and (b) $gof(x)$

(vi) What can you say about the two composition functions in part (iv) above. Justify your answer.

(06) (i) If p and q are two propositions , give the truth tables for Conjunction , Disjunction , Conditional implication and Bi-conditional.

(ii) Using part (i) construct the truth tables and verify whether each of the following is a tautology, a contradiction or a contingency. Justify your answers.

(a) $(p \wedge q) \wedge \sim(p \vee q)$

(b) $(p \vee \sim q) \rightarrow (p \wedge q)$

(c) $(p \rightarrow q) \leftrightarrow (p \wedge \sim q)$