The Open University of Sri Lanka

Continuing Education / Stand Alone Courses in Science

Final Examination 2006/2007

PSE 3117 - Mathematics for Chemistry and Biology

(2 hours)

> - This paper has **eight(8)** questions. Total marks awarded = 120
> - Attempt all the questions; all those scoring about 100 or more would be deemed to have scored 100%; the marks for the others would correspond to the actual mark obtained for this paper.
> - The use of a non-programmable electronic calculator is permitted.
> - Clearly write down all relevant steps in answering the questions.

1.(a) Simplify:

(i) $(a - 2b)^2 + 3a^2 - 3b^2 - (2a - b)^2$ (ii) $\dfrac{a^3 \times (a^2)^2 \times (2^4)^2}{a^5 \times 16 \times 8} - (a-1)^2 - (2a-1)$

(06 marks)

(b) Solve:

(i) $16x^2 - 24x + 5 = 0$ (ii) $64 - 225\, x^2 = 0$ (iii) $\log_3(x) + \log_3(3x) = 3$

(07 marks)

2.(a) Evaluate (**without the aid of a calculator or log tables**), the following:

$$\frac{\log_2 8 + \log_3 81}{\log_4 64} - \frac{1}{3}\log_2 125 \times \log_5 2$$

(03 marks)

(b) Deduce whether the following number is real or complex

$$\frac{(2 - i)}{(2 + i)} + \frac{3 + 2i}{2} - \frac{i}{5}$$

(04marks)

(c)   (i) Prove that $\cos 3\theta = 4\cos^3 \theta - 3\cos \theta$

(ii) Express $\cos(A - B)$ in terms of $\cos A$, $\cos B$, $\sin A$ and $\sin B$ only.

(iii) Show that the value of $\sin 2\theta (\tan \theta + \cot \theta)$ is independent of θ and is equal to an integer; find the value of this integer.

(09 marks)

(d) Given that $\sin 30° = \dfrac{1}{2}$, and $\sin 45° = \dfrac{1}{\sqrt{2}}$, evaluate $\sin 75°$ and $\cos 15°$

(04 marks)

3. (a) Differentiate (with respect to x) $y = (x-1)^2$ **from first principles**

(04 marks)

(b) Differentiate the following with respect to x.

(i) $y = \tan x - \sec x$ (ii) $u = \sqrt{(5-x)} + \dfrac{1}{2x^2}$ (iii) $y = \dfrac{x^2 - 2}{x+1}$

(12 marks)

4. The gradient of a curve (of the form $y = f(x)$) is given as $\dfrac{dy}{dx} = 3(x+1)(x-2)$. This curve has two turning points, one of which is at $(-1, \dfrac{9}{2})$.

(i) Find the equation of this curve.

(ii) Determine the other turning point.

(iii) Identify these turning points as maxima, minima or points of inflexion.

(09 marks)

5. (a): Integrate the following:

(i) $\int (3 - 2\sqrt{x} + 5x^{-2}).dx$ (ii) $\int \dfrac{\cos\theta}{\sin\theta + 1} d\theta$ (iii) $\int x.\ln x.dx$

(12 marks)

(b) Evaluate the following integrals

(i) $\dfrac{1}{4.606} \int\limits_{10}^{100} \dfrac{1}{x}$ (ii) If $\int\limits_{0}^{\frac{\pi}{2}} A(\sin\theta + \cos\theta)d\theta = 1$ (A is a constant), find the value of A.

(08 marks)

6 (a) Given that $\varphi = f(r, \theta, \phi) = r\cos\theta\sin\phi$,

(i)   determine all three first partial differentials.

(ii)   hence, write down the expression for $d\varphi$, the total differential.

(iii) Prove that $\left[\dfrac{\partial}{\partial\theta} \left(\dfrac{\partial\varphi}{\partial r} \right)_{\theta,\phi} \right]_{r,\phi} = \left[\dfrac{\partial}{\partial r} \left(\dfrac{\partial\varphi}{\partial\theta} \right)_{r,\phi} \right]_{\theta,\phi}$

(13 marks)

(b) Show that the following equation is an **exact differential** equation;

$$(uv - 2v^2)du + (\frac{1}{2}u^2 - 4vu)\,dv = 0$$

(03 marks)

7 (a) The following are the marks obtained by student **A** for chemistry in the first year, second year and third year (with weighting factors, 1,2,3 respectively) in the B.Sc degree programme.

1st year – 60; 2nd year – 65 ; 3rd year – 60

A second student, **B,** scores 40 in the 1st year, 60 in the 2nd year but, still, has an overall weighted average that is four(4) marks more than that of **A.** Determine the marks obtained by **B** in the third year.

(04 marks)

(b) In a packet of flower seeds, $\frac{1}{3}$ are known to correspond to yellow flowers and the remainder corresponds to red flowers.

(i) Calculate the probability of getting 0,1,2 yellow flowers in a row of five plants.
(ii) If 245 rows, each of five plants, are planted, approximately how many rows will contain all red flowers

(07 marks)

8. On repeating an experiment under identical conditions, the following 50 results were reported as the yield (in grams) of a certain product.

2.29	2.53	2.45	2.68	2.75	2.81	2.35	2.55	2.48	2.66	2.83	2.38
2.25	2.76	2.43	2.64	2.84	2.34	2.27	2.72	2.53	2.66	2.85	2.39
2.22	2.74	2.56	2.46	2.23	2.82	2.36	2.38	2.43	2.58	2.68	2.46
2.47	2.55	2.52	2.57	2.66	2.62	2.73	2.75	2.19	2.91	2.15	2.18
2.92	2.93										

i) Classify the above data into 9 classes of equal width, commencing with the class interval 2.11 – 2.20. (Any other classification **will not be** accepted)

ii) Construct the corresponding frequency table (include the class number, class interval, class width, class mark, tally mark and the class frequency); hence, draw the histogram and the frequency polygon.

iii) Calculate the classified mean.

(15 marks)

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