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The Open University of Sri Lanka
Department of Electrical and Computer Engineering
ECX6242 – Modern Control Systems
Final Examination – 2013/2014

Date: 2014-08-24 Time:0930-1230

Answer **five** questions by selecting **at least two questions** from each of the **sections A** and **B**.

Section A

Q1.

Consider the system represented in state variable form

$$\dot{x} = \boldsymbol{A}x + \boldsymbol{B}u$$
$$y = \boldsymbol{C}x + \boldsymbol{D}u$$

where

$\boldsymbol{A} = \begin{bmatrix} 1 & 1 \\ -5 & -10 \end{bmatrix}$, $\boldsymbol{B} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$, $\boldsymbol{C} = [6 \quad -4]$ and $\boldsymbol{D} = [0]$.

(a) Verify that the system is observable and controllable.

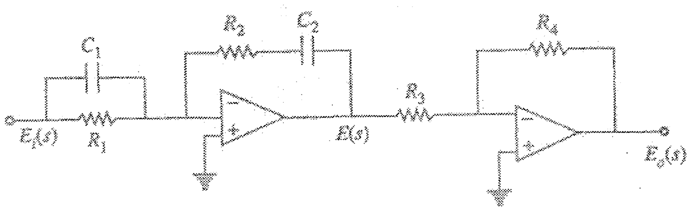
(b) If so, design a full-state feedback law and an observer by placing the closed-loop system poles at $s_{1,2} = -1 \pm j$ and the observer poles at $s_{1,2} = -10$.

Q2.

(a) Explain the structure of a PID controller.

(b) Briefly describe a PID controller tuning methodology.

(c) Show that the following circuit is a PID controller.



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Q3.

(a) Briefly describe Lyapunov's method for the determination of the stability of non-linear systems.

(b) An autonomous system is expressed as follows:

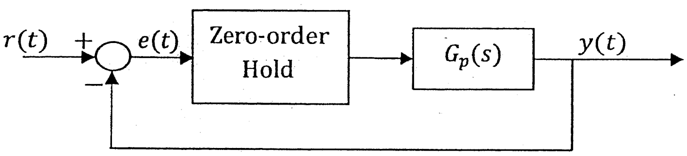
$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -m_1 x_2 - m_2 x_1$$

Study the stability of the system using Lyapunov's method considering the Lyapunov's function as $W = x_1^2 + x_2^2$.

Q4.

(a) Discuss some disadvategs of digital controller design.

(b) A system of the form shown in the figure has $G_p(s) = \frac{10}{s+1}$. Determine the range of sampling period T for which the system is stable. Select a smapling period T so that the system is stable and provides a rapid response.



Section B

The questions in this section are based on the paper reproduced at the end of this question paper. Devote at least half an hour to reading through the paper. Use your own words in your answers so as to demonstrate that you have understood the concepts described in the paper, do not copy extracts from the paper itself.

Q5. Briefly explain what are the issues found in multiple input multiple output control systems.

Q6. Explain the process of a predictive controller.

Q7. Describe the proposed methodology in your own words.

Q8. What are the advantages of the proposed method over PID controller?

State Space Representation of MIMO Predictive PID Controller

M.H.Moradi

Abstract— The paper is concerned with the state space representation of predictive PID controller, which has similar features to the model-based predictive controller (MPC). A MIMO PID type control structure is defined which includes prediction of the outputs and the recalculation of new set points using the future set point data. The optimal values of the MIMO PID gains are calculated using the values of gains calculated from an unconstrained generalised predictive control algorithm. The stability issues for this controller will be discussed. Simulation studies demonstrate the performance of the proposed controller and the results are compared with conventional PID and generalised predictive control solutions.

***Index Terms*— PID Control Design, Predictive PID Controller, MIMO Controller, State Space Representation.**

I. INTRODUCTION

The robustness and simplicity of Proportional-Integral-Derivative (PID) controllers has ensured the continued and widespread use these controllers in industry. For simple and undemanding processes, PID control yields satisfactory performance. But PID control is restricted in its performance for processes with time delay effects, non-minimum phase behaviour, unusual dynamics or a multivariable system structure. For these more complex control problems, advanced techniques such as generalized predictive control (GPC) or dynamic matrix control (DMC) may be required to achieve better control performance. For example, in the petrochemical industries, the GPC method (Clarke et al., 1987) has become one of the most popular Model Predictive Control (MPC) methods to be implemented. A common hurdle to successful implementation of advanced controllers at the Distributed Control System (DCS) level is the limited support in terms of hardware, software, and personnel training available within many industries. Plant personnel at the DCS level are frequently unable to provide the higher level of programming skill and the extra commitment needed to introduce advanced controllers and their associated features like failsafe redundancy. Implementing model-based control may also require capital investment to support new hardware and software products and also resources to train personnel in the operational behaviour of the new advanced controllers.

Meanwhile, the academic control community has developed many new techniques for tuning PID controllers. Often these methods try to stretch the capabilities of PID control to match the performance of the advanced controller designs.

This paper is concerned with the predictive PID method, which is approximating the control signal of advanced technique (GPC) by using a cost function. The method is called optimal PID control signal-matching method. It is an extension of the previously published SISO predictive PID controller (Moradi et al., 2002) (Katebi and Moradi, 2001) to MIMO systems with state space representation. The paper has been organised as follows: Section 2 describes the structure of

MIMO PID type predictive controller in state space representation. Section 3 presents a method to calculate the optimal values of the controller gain. In section 4, stability issues are discussed. A comparison between the proposed method and GPC technique is presented in Section 5. Finally, conclusions close the paper.

II. STATE SPACE REPRESENTATION OF THE MIMO PREDICTIVE PID CONTROLLER

The state space representation has the advantage that it can be used for multi-variable processes in a straightforward manner. MPC has been formulated in the state space context by (Morari 1994; Maciejowski 2001).

System Representation

Consider the deterministic state space representation of the plant as follows:

$$\begin{aligned} x_p(k+1) &= A_p x_p(k) + B_p \Delta u(k) \\ y(k) &= C_p x_p(k) \end{aligned} \qquad (1)$$

where:

A_p, B_p, C_p : Coefficients matrices

L: number of input and output

N: number of state

The system is assumed to be completely controllable and observable.

Output Prediction

The output of the model for step (k+i), assuming that the state at step k and future control increments are known, can be computed by recursively applying equation (1) resulting in (Camacho and Bordons, 1999):

$$y(k+1) = C_p A_p^2 x_p(k) +$$

$$\begin{bmatrix} C_p A_p^{i-1} B_p & C_p A_p^{i-2} B_p & \cdots & C_p B_p \end{bmatrix} \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \vdots \\ \Delta u(K+i-1) \end{bmatrix} \qquad (2)$$

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MIMO Predictive PID controllers Description

The standard digital control system is shown in Fig 1. In this unity feedback structure the process is an L- square multivariable system. The conventional MIMO PID controller in discrete form can be represented by:

$$u(k) = K_P e(k) + K_I \sum_{j=1}^{k} e(j) + K_D[e(k) - e(k-1)]$$

The incremental MIMO PID controller can be given by:

$$\begin{aligned} \Delta u(k) &= u(k) - u(k-1) = (K_P + K_I + K_D)e(k) \\ &\quad + (-K_P - 2K_D)e(k-1) + K_D e(k-2) \\ \Delta u(k) &= u(k) - u(k-1) = (K_P + K_I + K_D)e(k) \\ &\quad + (-K_P - 2K_D)e(k-1) + K_D e(k-2) \end{aligned} \tag{3}$$

In compact matrix form, equation (3) can be written as:

$$\Delta u(k) = KE(k) = K[R(k) - Y(k)] \tag{4}$$

where:

$$\Delta u(k) = [\Delta u_1(k) \;\; \Delta u_2(k) \;\; \cdots \;\; \Delta u_L(k)]^T$$

$$K = [K_D \;\; -2K_D - K_P \;\; K_D + K_P + K_I]$$

$$Y(k) = \left[y(k-2)^T \;\; y(k-1)^T \;\; y(k)^T\right]^T$$

$$y(k) = [y_1(k) \;\; y_2(k) \;\; \cdots \;\; y_L(k)]^T$$

$$E(k) = \left[e(k-2)^T \;\; e(k-1)^T \;\; e(k)^T\right]^T$$

$$e(k) = [e_1(k) \;\; e_2(k) \;\; \cdots \;\; e_L(k)]^T$$

$$R(k) = \left[r(k-2)^T \;\; r(k-1)^T \;\; r(k)^T\right]^T$$

$$r(k) = [r_1(k) \;\; r_2(k) \;\; \cdots \;\; r_L(k)]^T \qquad K \in R^{L\times 3L}$$

$$R, Y, E \in R^{3L\times 1} \qquad r, e, y, \Delta u \in R^{L\times 1}$$

and $K_P, K_I, K_D \in R^{L\times L}$ are the proportional, integral and derivatives gain matrices, respectively.

Using equation (4) a predictive PID controller is defined as follows:

$$\begin{aligned} \Delta u(k)_{PPID} &= K\sum_{i=0}^{M} E(k+i) \\ &= K\sum_{i=0}^{M} R(k+i) - K\sum_{i=0}^{M} Y(k+i) \end{aligned} \tag{5}$$

where:

PPID stands for Predictive PID

$\Delta u(k)_{PPID}$ Incremental form of control signal of the Predictive PID

~~The controller consists of M parallel PID controllers.~~ For M=0, the controller is identical to the conventional PID in equation (4). For M>0 the proposed controller has predictive capability similar to MPC where M is prediction horizon of PID controller. The controller signal in equation (5) can be decomposed as:

$$\begin{aligned} \Delta u(k)_{PPID} &= K[E(k) + E(k+1) + \ldots + E(k+M)] \\ &= \Delta u(k) + \Delta u(k+1) + \ldots + \Delta u(k+M) \end{aligned} \tag{6}$$

The input of i th PID at time k depends on the error signal at time (k+i). This implies that the current control signal value is a linear combination of the future predicted outputs. Therefore, the future outputs for M step ahead needs to be predicted.

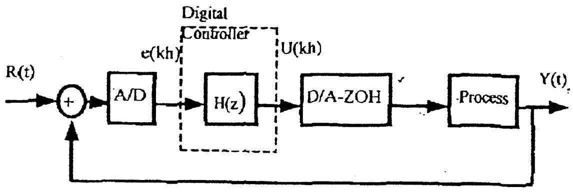

Fig.1: The closed loop block diagram for digital process control.

Lemma 1:

Using the model equation (1) and the predictive PID controller equation (5), the control signal of Predictive PID will be:

$$\Delta u_{PPID}(k) = (I + KH_t)^{-1}[KR_t(k) - KF_t x_p(k)] \tag{7}$$

where:

$$H_t = \sum_{i=0}^{M} H_i' \quad R_t(k) = \sum_{i=0}^{M} R(k+i) \quad F_t = \sum_{i=0}^{M} F_i$$

Proof: [See Moradi thesis (2002)]

In equation (7) the control signal of MIMO predictive PID controller is defined as function of process specifications (H_i', F_i) using equation A4, future set point information Rt and the PID gains.

III. Optimal Values of the Predictive PID Gains

To obtain the optimal values of the gains for the PID controller, the GPC algorithm is used as the ideal solution. For typical process control problems the default setting of output cost horizon $N_1 : N_2 = 1 : N$, and the control cost horizon $N_u = 1$ can be used in GPC to give reasonable performance (Clarke et al., 1987). This leads to a GPC control increment of [Camacho and Bordons, 1999]:

$$\Delta u_{GPC}(k) = K_{GPC}W(k) - K_0 x_p(k) \tag{8}$$

where:

$$K_{GPC} = [H^T H + \lambda I]^{-1} H^T \qquad K_0 K_{GPC} F$$

$$W(k) = \left[w(k)^T \;\; w(k+1)^T \;\; \cdots \;\; w(k+N)^T\right]^T$$

$$w(k) = [w_1(k) \;\; w_2(k) \;\; \cdots \;\; w_L(k)]^T$$

$$H = \begin{bmatrix} B_p & 0 & \cdots & 0 \\ A_p B_p & B_p & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A_p^{N-1} B_p & A_p^{N-2} B_p & \cdots & B_p \end{bmatrix} \qquad F = \begin{bmatrix} A_p \\ A_p^2 \\ \vdots \\ A_p^{N-1} \end{bmatrix}$$

The predictive PID control gains are then chosen to minimize the norm difference between the predictive PID signal, equation (7), and the GPC controller signal, equation (8). This means the following optimisation problem is to be solved:

$$Min_{K \in K_{PID}^S, M} J(K, K_0) = \| \Delta u_{PPID}(K) - \Delta u_{GPC}(K_0) \|_2$$

(9)

and K_{PID}^S = Set of stability gain for PID

The PID control signal (7) and the GPC control signal (8) both depend on the matrix quantity Z, where:

$$Z = x_p(k)$$

But the Z used in each control law will be a function of the sequence of past control gains used in each case. Thus this is a non-linear relationship. To make the analysis tractable, it is assumed that Z arising from the application of predictive PID law evolve closely through the optimal controls and outputs generated by the GPC algorithm, so that Z is written to first order as:

$$Z(K) = Z(K_0) + \Delta Z$$

With this assumption the following result can be established for the optimization problem (9):

Theorem 1

Given the predictive PID control law (7), and the GPC control law (8). If it is assumed that

$$Z_{PPID} = Z(K_0) + \Delta Z$$

then

$$J(K, K_0) = \| \Delta u_{PPID}(K) - \Delta u_{GPC}(K_0) \|_2 \le \| -(I + KH_t)^{-1} KF_t \|_2 \|\Delta Z\|_2$$

with

$$(I + KH_t)^{-1} KR_t(k) = K_{GPC} W(k)$$
$$(I + KH_t)^{-1} KF_t = K_0 \qquad (10)$$

where: $\|\Delta Z\|_2$ is suitably small.

Proof: [See Moradi thesis (2002)]

The solution for K will be found in terms of from second equation in (10):

$$K_0(I + KH_t) = KF_t \rightarrow K(F_t - H_t K_0) = K_0 \quad (11)$$

A unique solution to equation (11) always exists and takes the form (Levine, 1996):

$$K = K_0 (F_t - H_t K_0)^T [(F_t - H_t K_0)(F_t - H_t K_0)^T]^{-1} \quad (12)$$

From first equation in (7) the rebuilt future set point will be calculated as:

$$R_t(k) = K^{-1}(I + KH_t) K_{GPC} W(k) \qquad (13)$$

The predictive PID controller can be implemented using the following procedure.

Algorithm 1: Predictive PID controller for state space process model representation.

Step 1: Initialisation

1. Find a system model and calculate the discrete matrices, A_p, B_p, C_p.

2. Choose the value of prediction horizon, M, and formulate the future set point vectors W

Step 2: Off line Calculation

1 Calculate the matrices H_t, F_t in equation (A6) using equation (A4)

2 Calculate the GPC gain, K_{GPC}, using equation (8)

3 Calculate the optimal value of predictive PID gains using equation (12)

4 Iterate over the value of M to minimize the cost function.

Step 3: On line Calculation

1 Calculate the following signals

a $F_t Z(K)$

b $R_t(k)$ using equation (13)

2 Calculate the control increment

$$u(k) = u(k-1) + (I + KH_t)^{-1} K[R_t(k) - F_t Z(K)]$$

Step 4: Assessment

1 Apply the control signal.

2 Check closed loop performance.

IV Stability Issues for Predictive PID Controllers

In this section, first, the closed loop state space representation of conventional PID is presented and then the closed loop representation of proposed method will be found. This closed loop representation will be used for stability studies of method.

Conventional MIMO PID Closed Loop System

The discrete state space representation of conventional MIMO PID controller is:

$$x_c(k+1) = A_c x_c(k) + B_c e(k)$$
$$\Delta u(k) = C_c x_c(k) + D_c e(k) \qquad (14)$$

where A_c, B_c, C_c, D_c are coefficient matrices

Using the state space model of system, equation (1), the composite closed loop state space system can be written as:

$$\begin{bmatrix} x_p(k+1) \\ x_c(k+1) \end{bmatrix} = A_{cl} \begin{bmatrix} x_p(k) \\ x_c(k) \end{bmatrix} + \begin{bmatrix} B_c D_c \\ B_c \end{bmatrix} u(k) \quad 15$$

The eigenvalues of matrix are the poles of the closed loop system, where:

$$A_{cl} = \begin{bmatrix} A_c - B_p D_c C_p & B_p C_c \\ -B_c C_p & A_c \end{bmatrix}$$

Predictive PID Closed Loop System

Despite the simplicity of the, conventional MIMO PID controller equation (14), the calculation of the state space representation of MIMO predictive PID controller is not straightforward, but it can be found using the following lemma:

Lemma 2:

Using the state space representation of conventional PID controller equation (14) and the predictive PID controller equation (5), the state space representation of MIMO predictive PID controller can be shown to be:

$$x_c(k+1) = A_{cc} x_c(k) + B_{cc} \sum_{i=0}^{M} r(k+i) - A_{cp} x_p(k)$$
$$\Delta u(k) = C_{cc} x_c(k) + D_c \sum_{i=0}^{M} r(k+i) - D_{cp} x_p(k) \qquad (16)$$

Proof: [See appendix B]

Using equation (3) as model of system and equation (16) as controller, the closed loop state space system incorporating predictive PID control can be written:

$$\begin{bmatrix} x_p(k+1) \\ x_c(k+1) \end{bmatrix} = \begin{bmatrix} A_{cp} - B_p D_{cp} & B_p C_{cc} \\ -A_{cp} & A_{cc} \end{bmatrix} \begin{bmatrix} x_p(k) \\ x_c(k) \end{bmatrix} \quad (17)$$

To check the stability of system, all the eigenvalues of the closed loop system matrix, A_{cl}, should lie inside the unit circle, where:

$$A_{cl} = \begin{bmatrix} A_{cp} - B_p D_{cp} & B_p C_{cc} \\ -A_{cp} & A_{cc} \end{bmatrix}$$

In next section several different systems will be considered and the stability region for those systems will be found using closed loop system matrix equation (17).

V. Case Studies:

In this section, the stability regions of predictive PID and performance comparison of proposed method and GPC for two industrial systems will be discussed ,Fig 3, the systems are:

1) The small signal model of a stirred tank reactors: (Camacho and Bordons, 1999).

where the manipulated variable $u_1(s)$ and $u_2(s)$ are the feed flow rate and the flow of coolant in the jacket respectively. The controlled variables $Y_1(s)$ and $Y_2(s)$ are the effluent concentration and the reactor temperature respectively (Table1).

2 The boiler model (Moradi et al, 2002)

where the manipulated variable $u_1(s)$ and $u_2(s)$ are the feed/air demand and the control valve position respectively. The controlled variables $Y_1(s)$ and $Y_2(s)$ are throttle pressure and the steam flow respectively (Table1).

.*Stability Study*

The effect of M on predictive PID parameters and variation of predictive PID parameters on stability region has been considered for Systems G1 and G2.

It has been found empirically that a larger M value decreases the predictive PID coefficient and for a stable system (G1) increases the size of stability region, Fig 4a and for a non-minimum phase system (G2) a larger M decreases the size of stability region, Fig 4b.

.*Performance Study*

GPC and predictive PID methods were used to design the controller for systems G1 G2. For GPC, the horizon prediction of output N=20, control input horizon Nu=1 were assumed. The controller gains for the two methods are shown in Table 1.

It is clear from the Table 1 that for the first order 2I2O system (G1) conventional PID is enough to achieve the GPC performance (M=0). The step response of the closed loop system of G1 for two methods has been shown in Fig 5.

For the boiler model, M=1 is sufficient to approximate the GPC performance. The step response of the closed loop system for two methods along with conventional MIMO PID (Maciejowski's method, Katebi et al, (2000)) is shown in Fig 6. The results show that the predictive PID control performance is close to that of the GPC controller and is superior to the conventional PID control.

Fig. 7 shows the simulation result for the case which where future set point is not known. The predictive PID response is almost critically damped compared to PID where the response is over damped.

Remark

It has been found that for a first order 2I2O system, conventional PID (predictive PID with M=0) is sufficient to achieve the GPC performance. The variation in the time step responses for different values of M shows that this parameter can be used to meet some additional time domain control design specifications.

VI Conclusions

In this paper, the state space representation of MIMO predictive PID controller design was described, which is an optimal PID control signal matching method. The method needs a lot of calculation because the designer has to solve an advanced control method, but the controller has PID structure so, it can be applied to the system easily. The proposed controller can deal with future set points and the process dead time can be incorporated without any need to approximation. The controller reduces to the same structure conventional PID controller for first order 2I2O system. It was shown that the optimal values of PID gains could be found using a scheme similar to a MPC. Various benchmark processes were employed to illustrate the stability and performance of the proposed method. One of the main advantages of the proposed controller is that it can be used with systems of any order and the PID tuning can be used to adjust the controller performance.

VI. Acknowledgement

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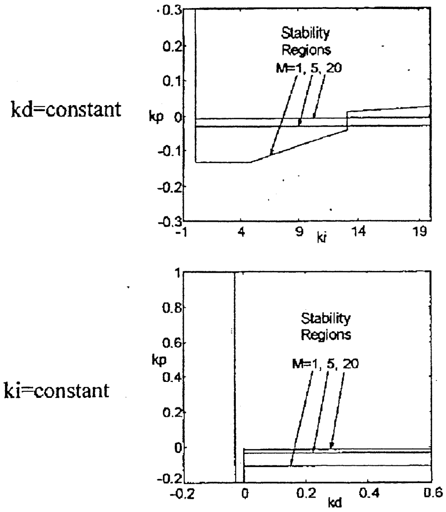


Fig 2a: The stability Regions of Predictive PID method for the small signal model of a stirred tank reactor.

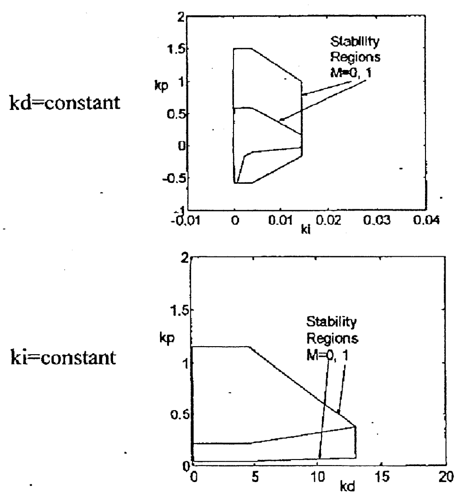
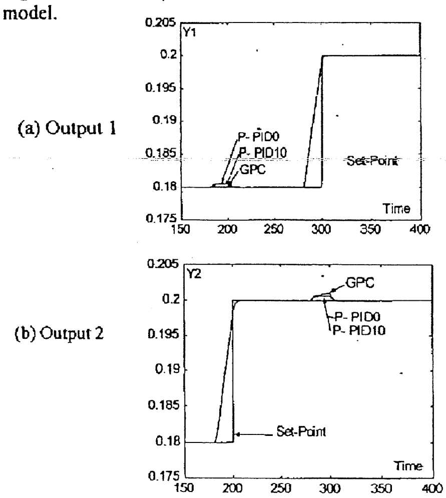


Fig 2b: The stability Regions of Predictive PID method for Boiler model.



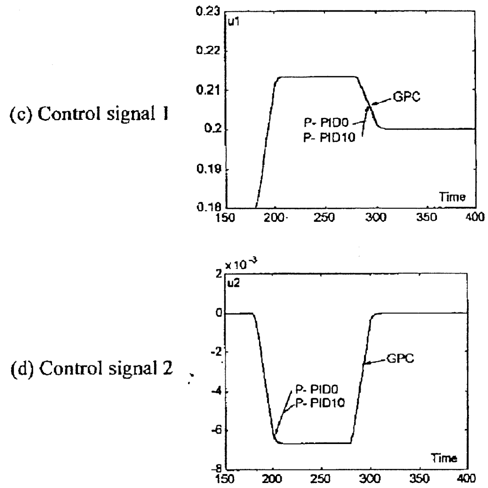
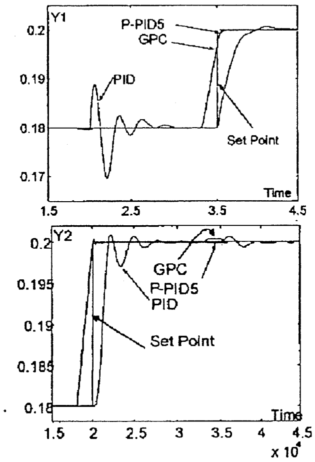


Fig.3: The comparison of GPC method with Proposed predictive PID Method for A stirred tank reactor.



P-PID5: Predictive PID controller with M=5

Fig. 4: The comparison of Predictive PID method with GPC and conventional PID methods for boiler.

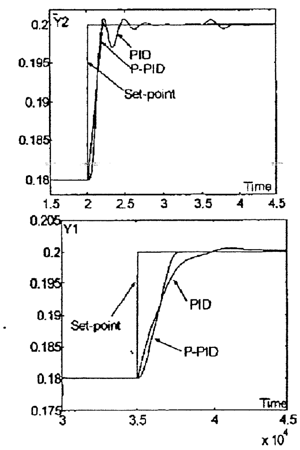


Fig 5: The comparison of predictive PID method with conventional PID without future set point information