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THE OPEN UNIVERSITY OF SRI LANKA

Department of Electrical and Computer Engineering
Final Examination 2013 /2014

ECX 6234 – Digital Signal Processing

Closed Book Test

Time 09:30 -12:30 **Date: 16 August 2014**

Answer five questions selecting at least one question from section B.

Section A

Question 1

(a) Briefly, outline four advantages and one disadvantage of digital signal processing (DSP) as opposed to analogue signal processing.

[5 marks]

(b) Define each of the following terms as applied to discrete time signal processing systems:

(i) Linearity
(ii) Time-invariance
(iii)Causality

[3 marks]

(c) A system is defined by the input output relation as,

$$y[n] = x[n] + 4x[n-1]$$

Find the system is,

(i) Linear
(ii) Time – invariant
(iii) Causal

[3 marks]

(d) (i) What is BIBO stability?

[3 marks]

(ii) Show that the necessary and sufficient condition for a LTI system to be BIBO stable is

$$\sum_{n=-\infty}^{\infty} |h(n)| \le M_h \le \infty$$

For some constant M_h.

[3 marks]

(iii) Determine whether the following system characterized by its impulse response is BIBO stable.

$$h[n] = (-5)^n u[n]$$

[3 marks]

Question 2

(a) Define the Z-transform of a discrete sequence $x[n]$.

[2 marks]

(b) A speaker's manufacturer has developed a LTI model for one of its product and its transfer function is given below.

$$H(z) = \frac{z\,(z + 2.1)}{(z - 0.2)(z + 0.6)}$$

(i) Draw the poles and zeros of this model $H(z)$ on a plot in the z – domain.
(ii) Comment on general characteristics of the system that can be deduced from the plot.
(iii)Using partial fraction expansion, find a closed – form expression for the impulse response of the loudspeaker modeled by $H(z)$

[10 marks]

(c) Determine the Z transform H(z) of the impulse response h(n) of the following difference equations:
(i) $y[n] = x[n] + 2x[n-1] - 3x[n-2]$
(ii) $y[n] = x[n] + 0.2x[n-1] - 0.5y[n-1]$

[4 marks]

(d) If x[n] is given by $x[n] = \delta[n] - 0.5\delta[n-1]$, by using the inverse Z-transform methods, determine the output $y[n]$ for the difference equations in (c) above.

[4 marks]

Question 3

(a) Define the Discrete Fourier Transform (DFT) and explain how it is related to the Discrete Time Fourier Transform (DTFT).

[4 marks]

(b) (i) For the signal $x[n] = 0.8^n u[n]$, find $DTFT\{\,x[n]\}$
(ii) Find $DFT\{\,x[n]\}$ if $x[n] = \{1,1,-1,-1\}$

[6 marks]

(c) Let $z[n]$ represent the circular convolution of two sequences $x[n]$ and $y[n]$, given by

$$z[n] = x[n] \circledast y[n]$$

Using appropriate properties of the DFT, prove that

$$Z[k] = X[k]Y[k]$$

Where$Z[k]$, $X[k]$ and $Y[k]$ are the DFT of $z[n]$, $x[n]$ and , $y[n]$ respectively.

[4 marks]

(d) Compute the circular convolution of the following discrete – time sequences:

$$x[n] = \{2,3,1,5\}, \qquad y(n) = \{1,4,-3,-5\}$$

[6 marks]

Question 4

(a) Consider linear difference equation $y[n] + y[n-1] + 5x[n] - 3x[n-2] = 0$.
Show the realization of the output $y[n]$ on block diagram.
Rename $x[n-1]$ as $S_1[n]$ and $y[n-1]$ as $S_2[n]$. Express $S[n+1]$ in the form

$$s[n+1] = [A]s[n] + [B]x[n]$$

$$y[n] = [C]s[n] + [D]x[n]$$

$s[n+1]$= state vector $y[n]$ = Output vector

$[A], [B], [C]$ & $[D]$ are matrices of appropriate dimensions.

[12 marks]

(b)

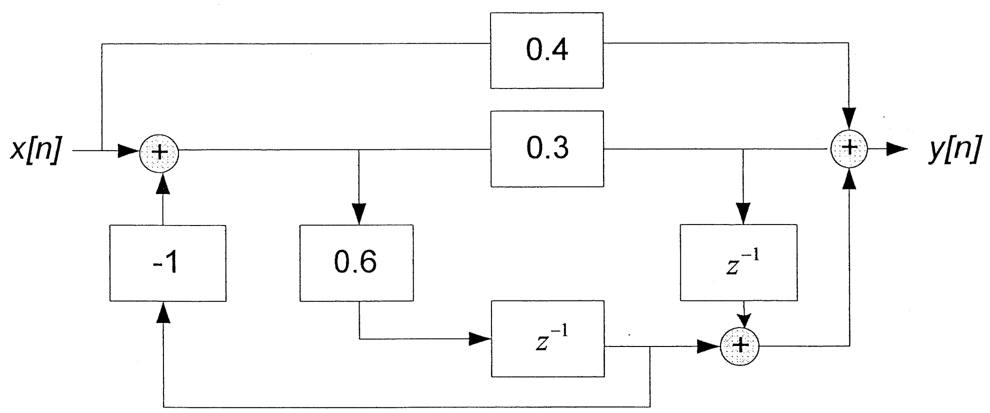

Figure 4

(i) Find the transfer function $H(z) = \frac{Y(z)}{X(z)}$

(ii) Find the impulse response $h(n)$

[8 marks]

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Question 5

a) Explain why analogue signals are generally low-pass filtered before they are converted to digital form.

[4 marks]

b) A continuous - time signal $x_a(t)$ with bandwidth ***B*** and its echo $x_a(t-\tau)$ arrive simultaneously at a TV receiver. The received analog signal given by,

$$s_a(t) = x_a(t) + \alpha x_a(t-\tau) \quad |\alpha| < 1$$

This signal processed by the system shown in figure 5.

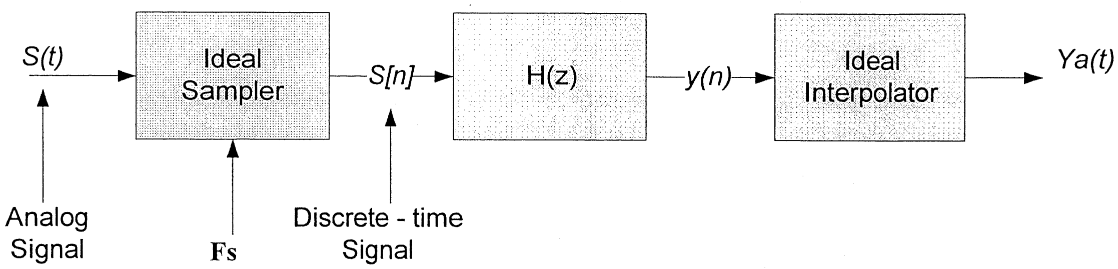

Figure 5

It is needed to remove "ghost" signal ($x_a(t-\tau)$) from the receiving signal. Specify $H(z)$ so that $y_a(t) = x_a(t)$.

[16 marks]

Section B

Question 6

(a) Sketch the impulse response of an ideal low pass filter $h_d(n)$.

[2 marks]

(b) In order to realize the above filters as a FIR filter, it is necessary to truncate the desired impulse response $h_d(n)$.

Therefore, the impulse response of the FIR filter can be written as

$$h(n) = h_d(n)w(n)$$

(i) What is the function of $w(n)$

(ii) If $w(n) = 1, \; 0 \leq n \leq L$

Sketch $W(\omega)$. Write the relationship between $H(\omega)$, $H_d(\omega)$and $W(\omega)$.

[2 marks]

(c) Design a linear low pass filter using window techniques, to meet the following requirements.

Pass band cut off frequency = 4 kHz
Stop band cut off frequency = 5 kHz
Attenuation = - 40dB
Sampling Frequency = 48kHz

[16marks]

Question 7

(a) What is meant by
(i) Up sampling
(ii) Down sampling

Of a digital signal?

[6 marks]

(b) A signal $x[n]$ can be sampled in to $v[n]$ using the operator $\delta_D[n]$ as follows:
$v[n] = \delta_D[n] \, . \, x[n]$ where $\delta_D[n] = 1$ if n/D is an integer.
= 0 otherwise

(i) Write an expression for $\delta_D[n]$ if D =2.
(ii) Show that $V(z) = \frac{1}{2} X(z) + \frac{1}{2} X(-z)$ for the case given in (i)

[8 marks]

(c) Figure 7 shows a downsampler followed by an upsampler.

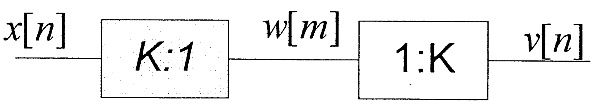

Figure 7

Show that $V(z) = W(z^K)$

[6 marks]

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Question 8

(a) Give a block diagram of Kalman filter and briefly explain the principle of operation.

[8 marks]

(b) Briefly explain the following

- (i) Multistage implementation of digital filters.
- (ii) Prediction based sampling method.
- (iii) Fast Fourier transforms algorithms.

[12 marks]

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Supplementary materials

Window type	*Window function* $w(n)$ $(0 \le n \le N-1)$	Δw	*Attenuation*
Rectangular	1	$\frac{4\pi}{N}$	-13dB
Bartlett	$\frac{2}{N-1}\left(\frac{N-1}{2}-\left\lvert n-\frac{N-1}{2}\right\rvert\right)$	$\frac{8\pi}{N}$	-27dB
Hann	$0.5-0.5\cos\left(\frac{2\pi n}{N-1}\right)$	$\frac{8\pi}{N}$	-32dB
Hamming	$0.54-0.46\cos\left(\frac{2\pi n}{N-1}\right)$	$\frac{8\pi}{N}$	-43dB
Blackman	$0.42-0.5\cos\left(\frac{2\pi n}{N-1}\right)+0.08\cos\left(\frac{4\pi n}{N-1}\right)$	$\frac{12\pi}{N}$	-53dB