THE OPEN UNIVERSITY OF SRI LANKA

BACHELOR OF INDUSTRIAL STUDIES

TTZ5244 – QUANTITATIVE TECHNIQUES

FINAL EXAMINATION 2013/2014

DURATION – THREE HOURS

Date: 04th September 2014   **TIME: 0930 - 1230 Hours**

This question paper consists of three sections A, B, and C.

Answer all questions in Section A and two (02) questions each from sections B and C

Total questions to be answered are five (05)

You should clearly show the steps involved in solving problems.

No marks are awarded for the mere answers without writing the necessary steps

SECTION-A

This section carries 20 marks. Answer all the questions in this section.

(1) Solve the following equations

    (a) $3^{2x+1} \times 27^x = 729$ (b) $7^{2x+5} + 11 = 2412$ **(02 Marks)**

(2) Express the following expression with the x as the subject.

    (a) $y = A(1+y)^{4x}$ (b) $(4+y)2^{3x} = 6$ **(02 Marks)**

(3) What do you understand by $\left[\dfrac{dy}{dx}\right]$, **if y** is a function of x.   **(02 Marks)**

(4) Determine the second derivative of the following functions with respect to x

    (a) $y = 8x^5 + 3x^4 + 5x^3 + 4x^2 - 21$ (b) $y = x(4x^2 + 6)$ **(04 Marks)**

(5) Determine the stationary points of the following functions and find out whether they are
minima or maxima **(04 Marks)**

    (a) $y = 12x^3 - 4x^2 + 5$ (b) $y = 3x^3 - 3x - 2$

(6) Give an example for a symmetric matrix. **(02 Marks)**

(7) Define the "**inverse**" of a matrix. **(02 Marks)**

(8) Find the determinant of the matrix A, if **(02 Marks)**

$$A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & 4 \\ 1 & 2 & 1 \end{pmatrix}$$

SECTION B

Maximum possible mark for this section is 40. Answer any two questions from this section

(09) (a) What are the two important parameters in a straight line graph? **(02 Marks)**

    (b) Desktop publishing cost for a Statistics book amount to Rs. 2,000/=. In addition, Rs. 4/= per copy is incurred for printing the book. The publisher receives Rs 20/= a copy from the sale and pay 10% of this to the author.

      i. Write down an expression for publisher's total profit in terms of the number of copies printed (X). Assume that all copies are sold. **(04 Marks)**

      ii. Sketch a graph Profit (P) versus number of copies printed(X). **(05 Marks)**

    (c) An iso-cost curve shows the different combinations of two raw materials that can be purchased with the given budget for a factory. ABC factory allocates Rs. 6,000/= per day for purchasing raw material A and raw material B. Cost per item A and B is Rs. 20/= and Rs. 40/= respectively. Let the number of A items purchased is X and number of B items purchased is Y

      i. Obtain an equation involving X and Y. **(04 Marks)**

      ii. Sketch the iso-cost curve. (Graph of X and Y.) **(05 Marks)**

(10) (a) Differentiate the following functions with respect to X

   i. $y = e^{3x+4}$

   ii. $y = \log_e (4X^2 + 3X + 7)$ (04 Marks)

(b) The term "marginal revenue" in economics is defined as $\dfrac{dR}{dQ}$ where R is the Revenue and Q is the demand. If the revenue R and demand Q are related by, R = $10Q - 0.001Q^2$

What is the value of marginal revenue, when Q = 3000   (08 Marks)

(c) Price elasticity of demand E is defined as,

$$E = -\ \frac{P}{Q}\ \frac{dQ}{dP}$$ where Q is the demand and P is the price

If the demand function is $Q = 200 - P^2 - 6P$, what is the price elasticity of demand (E) when P = 5 ?   (08 Marks)

(11)   (a).   If A $= \begin{pmatrix} 4 & 2 & 0 \\ 2 & 3 & -1 \end{pmatrix}$ B $= \begin{pmatrix} 2 & 0 \\ 3 & 0 \\ 0 & 2 \end{pmatrix}$

C $= \begin{pmatrix} 4 & 0 \\ 2 & 0 \end{pmatrix}$ D $= \begin{pmatrix} 2 & 0 \\ 1 & 1 \\ 0 & 2 \end{pmatrix}$

Calculate following matrix products **where they exist**. (If the matrices do not exist describe the reasons)

        (i) AB   (ii) CD   (06 Marks)

(b).   A $= \begin{pmatrix} 1 & 0 \\ 2 & 4 \end{pmatrix}$

Calculate A^2 and A^3 (06 Marks)

(c).   Let A $= \begin{pmatrix} 3, & 1 \\ -1, & 2 \end{pmatrix}$, I $= \begin{pmatrix} 1, & 0 \\ 0, & 1 \end{pmatrix}$ and O $= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

Show that $A^2 - 5A + 7I = O$ (08 Marks)

(12) (a) Determine the inverse of the matrix   **(08 Marks)**

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$$

(b) Hence solve following sets of equations.

$X + Y + Z = 7$

$X + 2Y + 3Z = 4$

$X + 4Y + 9Z = 6$ **(12 Marks)**

SECTION C

Maximum possible mark for this section is 40. Answer any two questions from this section

(13). ABC company has two factories that ship to three regional warehouses. The cost of transportation per unit is given below.

Warehouse	Transportation costs (Rs) Factory	
	F_1	F_2
W_1	2	4
W_2	2	2
W_3	5	3

Factory F_2 is old and has a variable manufacturing cost of Rs 20/= per unit. Factory F_1 is modern and manufactures the products for Rs 10/= per unit. Factory F_2 has a monthly capacity of 250 units. And Factory F_1 has a monthly capacity of 400 units. The requirements at the warehouses are:

Warehouse	Requirement
W_1	200
W_2	100
W_3	250

How should each factory ship to each warehouse in order to minimise the total cost? Formulate this problem as a liner programming model.   **(20 marks)**

(14) A company produces two different products A and B. The profit contribution of A is Rs.160/= per unit and B is Rs 100/= per unit. The resources required to manufacture one unit of each product are shown below.

	Product A	Product B
Raw materials	4	2
Labour units	3	3
Machine time	2	5

The company has a daily supply of 24 units of raw materials, 21 units of labour and 30 units of machine time. The company management would like to determine how many units of each product should be produced in order to get the maximum profit.

(a) Name the variables in this problem. **(02 marks)**

(b) What is the objective of this problem? **(02 marks)**

(c) What are the constraints of the problem? **(02 marks)**

(d) Solve the **formatted programme graphically** to determine how the factory should schedule production in order to maximise the profit. **(14 marks)**

(15) ABC Company wishes to plan its advertising strategy. There are two popular newspapers under consideration, Newspaper A and Newspaper B. News paper A has a reach of 2,000 potential customers and Newspaper B has a reach of 2,500 potential customers. The cost of advertising per once is Rs 400/= for A and Rs. 600/= for B. The company has a monthly budget of Rs 6,000/=. There is a requirement that the total reach for the income group under the Rs.20,000/= per annum should not exceed 4,000 potential customers. The reach in Newspaper A and Newspaper B for this group is 400 and 200 potential customers respectively.

(a) Formulate the problem as a Linear Programme model. **(06 Marks)**

(b) Determine using the **Simplex Method,** how many times should the company advertise in both the newspapers to maximize the total reach? **(14 Marks)**