THE OPEN UNIVERSITY OF SRI LANKA
BACHELOR OF TECHNOLOGY -LEVEL 05
FINAL EXAMINATION 2013/2014
MPZ 5230-Engineering Mathematics III

Date: 12-8-2014 Time duration: 930hrs-1230hrs

Instructions:

- Answer only six(06) Questions
- No. of pages in the paper-04
- State clearly any assumptions you required
- All symbols are in standard notations, unless they are defined
- Statistical table is required

1. (a) The upper bound of non-rectangular region S is given by $y = \beta_1(x)$ and lower bound is given by $y = \beta_2(x)$ where $a \leq x \leq b$.
Write down the steps to evaluate the surface integral over the region S, $\int f(x,y)dA$,where f is a continuous function of x and y and A is the area of the bounded region. [25%]

   (b) Evaluate the following integrals:

   i. The surface integral of the function $x^2 + y^2$ over a circle of radius a and centre at the origin.

   ii. $\iint_R e^{2x+3y}dxdy$ where R is the region bounded by $x = 0$, $y = 0$, and $x + y = 1$.

   iii. $\iint_R r^3 dr d\theta$ where R is the area bounded between the circles $r = 2\cos\theta$ and $r = 4\cos\theta$. [75%]

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2. (a) Evaluate:

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{1}{\sqrt{1-x^2-y^2-z^2}} dxdydz.$$ [30%]

(b) Show that the moment of inertia of the cylinder about the Z-axis is $\frac{9M}{2}$ where M is the mass of the cylinder. [35%]

(c) Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$ [35%]

3. (a) Evaluate $\iint_S \underline{A} \cdot \underline{n} ds$, where

i. $\underline{A} = z\underline{i} + x\underline{j} - 3y^2 z\underline{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$. [25%]

ii. $\underline{A} = \frac{1}{r}\underline{e}_r (r \neq 0)$, and S is the curved surface S of the cylinder $r = R$ between $z = 0$ and $z = 1$ and $\underline{n}$ is the unit outward normal to the surface S. [25%]

(b) Let $\underline{c} = c_1\underline{i} + c_2\underline{j} + c_3\underline{k}$ be a constant vector and $\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$ is a position vector whose magnitude is r.

i. Evaluate $\underline{\nabla} \cdot (\underline{c} \cdot \underline{r})$ and $\underline{\nabla} \times (\underline{c} \times \underline{r})$ [20%]

ii. Show that $\underline{\nabla}\left(\frac{\underline{c} \times \underline{r}}{r^n}\right) = 0$ for any value of n and hence deduce that $\underline{\nabla} \times \left[\underline{\nabla} \times \frac{\underline{c} \times \underline{r}}{r^n}\right] = \underline{0}$. [30%]

4. (a) State the Gauss' Divergence theorem. [10%]

(b) Verify the Gauss' theorem for the vector field for: $\underline{F} = (x^2 - yz)\underline{i} + (y^2 - zx)\underline{j} + (z^2 - xy)\underline{k}$ taken over the rectangular parallopiped $0 \leq x \leq a$, $0 \leq y \leq b$, $0 \leq z \leq c$. [30%]

(c) Verify the Stokes theorem for the vector field $\underline{F} = -y\underline{i} + x\underline{j}$ and the surface S consisty of the upper half-surface of the unit sphere $x^2 + y^2 + z^2 = 1$, $z \geq 0$. [30%]

(d) Evaluate the surface integral $\iint_s Curl\underline{F} \cdot \underline{n} ds$ by transforming it into a line integral where S is a part of the surface of the parabolaid $z = 1 - x^2 - y^2$, for which $z \geq 0$ and $\underline{F} = y\underline{i} + z\underline{j} + x\underline{k}$. [30%]

5. (a) Find the image of the rectangular region in z-plane bounded by straight lines $x = 1$, $x = 2, y = 1, y = 2$ under the mapping $w = \frac{1}{z}$. [20%]

(b) Find the region of convergence of the series $\sum_{n=1}^{\infty} \frac{(z+5)^5}{(3n+4)^3} \left(\frac{1}{\sqrt{2}}\right)^n$. [25%]

(c) Evaluate $\oint_C \frac{3z^2+z}{z^2-1}$, where c is the circle $| z - 1 |= 1$. [25%]

(d) Explain $f(z) = \frac{z}{(z-1)(z-2)}$ in a Laurent series for

i. $| z |< 1$.

ii. $1 <| z |< 2$. [30%]

6. The components of a moment of inertia tensor are given by $[T_{ij}] = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 4 & \sqrt{3} \\ 0 & \sqrt{3} & 6 \end{pmatrix}$

(a) Find the eigenvalues and the principle axes of the tensor. [25%]

(b) Express the tensor with respect to its principle axes. [30%]

(c) Write the equation of the transformation relating to the coordinates x_i determined by the principle axes. [25%]

(d) Verify the result in (b) by using the transformation law for second order tensor. [20%]

7. (a) Solve the following partial differential equation by separation of variables.

$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} + au = 0$ in $0 \leq x \leq \pi$, $t > 0$

$u(0,t) = \frac{\partial u(\pi, L)}{\partial x} = 0$ for $t \geq 0$

$u(x,0) = x(\pi - x)$ in $0 \leq x \leq \pi$ where a is a constant. [60%]

(b) Apply fourier sine transforamtion to solve:

$\frac{\partial V}{\partial t} = \lambda \frac{\partial^2 V}{\partial x^2}$, $x > 0$, $t > 0$

subject to $V(0,t) = v_0$ and $V(x,0) = 0$. [40%]

8. Two random samples drawn from two normal populations have the variable values as below:

Sample I	19	17	16	28	22	23	19	24	26			
Sample II	28	32	40	37	30	35	40	28	41	45	30	36

Obtain the estimate of the variance of the population and test whether the two populations have the same variance. [100%]

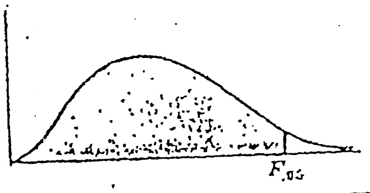
9. The advertising alternatives for a company include television, radio, and newspaper advertisements. The costs and estimates for audience coverage are given in table.

	Television	Newspaper	Radio
Cost per advertisement	$2,000	$600	$300
Audience per advertisement	100	40,000	18,000

The local newspaper limits the number of weekly advertisements from a single company to 10. Moreover, in order to balance the advertising among the three types of media, no more than half of the total number of advertisements should occur on the radio, and at least 10% should occur on television. The weekly advertising budget is $ 18,200.
How many advertisements should be run in each of the three types of media to maximize the total audience? [100%]

TABLE 50	95th PERCENTILE VALUES FOR THE F DISTRIBUTION

n_1 = degrees of freedom for numerator
n_2 = degrees of freedom for denominator
(shaded area = .95)



n_2 \ n_1	1	2	3	4	5	6	8	12	16	20	30	40	50	100	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	238.9	243.9	246.3	248.0	250.1	251.1	252.2	253.0	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.37	19.41	19.43	19.45	19.46	19.46	19.47	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.84	8.74	8.69	8.66	8.62	8.60	8.58	8.56	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.04	5.91	5.84	5.80	5.75	5.71	5.70	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.82	4.68	4.60	4.56	4.50	4.46	4.44	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.15	4.00	3.92	3.87	3.81	3.77	3.75	3.71	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.73	3.57	3.49	3.44	3.38	3.34	3.32	3.28	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.44	3.28	3.20	3.15	3.08	3.05	3.03	2.98	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.23	3.07	2.98	2.93	2.86	2.82	2.80	2.76	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.07	2.91	2.82	2.77	2.70	2.67	2.64	2.59	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	2.95	2.79	2.70	2.65	2.57	2.53	2.50	2.45	2.40
12	4.75	3.88	3.49	3.26	3.11	3.00	2.85	2.69	2.60	2.54	2.46	2.42	2.40	2.35	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.77	2.60	2.51	2.46	2.38	2.34	2.32	2.26	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.70	2.53	2.44	2.39	2.31	2.27	2.24	2.19	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.64	2.48	2.39	2.33	2.25	2.21	2.18	2.12	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.59	2.42	2.33	2.28	2.20	2.16	2.13	2.07	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.55	2.38	2.29	2.23	2.15	2.11	2.08	2.02	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.51	2.34	2.25	2.19	2.11	2.07	2.04	1.98	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.48	2.31	2.21	2.15	2.07	2.02	2.00	1.94	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.45	2.28	2.18	2.12	2.04	1.99	1.96	1.90	1.84
22	4.30	3.44	3.05	2.82	2.66	2.55	2.40	2.23	2.13	2.07	1.98	1.93	1.91	1.84	1.78
24	4.26	3.40	3.01	2.78	2.62	2.51	2.36	2.18	2.09	2.03	1.94	1.89	1.86	1.80	1.73
26	4.23	3.37	2.98	2.74	2.59	2.47	2.32	2.15	2.05	1.99	1.90	1.85	1.82	1.76	1.69
28	4.20	3.34	2.95	2.71	2.56	2.45	2.29	2.12	2.02	1.96	1.87	1.81	1.78	1.72	1.65
30	4.17	3.32	2.92	2.69	2.53	2.42	2.27	2.09	1.99	1.93	1.84	1.79	1.76	1.69	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.18	2.00	1.90	1.84	1.74	1.69	1.66	1.59	1.51
50	4.03	3.18	2.79	2.56	2.40	2.29	2.13	1.95	1.85	1.78	1.69	1.63	1.60	1.52	1.44
60	4.00	3.15	2.76	2.53	2.37	2.25	2.10	1.92	1.81	1.75	1.65	1.59	1.56	1.48	1.39
70	3.98	3.13	2.74	2.50	2.35	2.23	2.07	1.89	1.79	1.72	1.62	1.56	1.53	1.45	1.35
80	3.96	3.11	2.72	2.48	2.33	2.21	2.05	1.88	1.77	1.70	1.60	1.54	1.51	1.42	1.32
100	3.94	3.09	2.70	2.46	2.30	2.19	2.03	1.85	1.75	1.68	1.57	1.51	1.48	1.39	1.28
150	3.91	3.06	2.67	2.43	2.27	2.16	2.00	1.82	1.71	1.64	1.54	1.47	1.44	1.34	1.22
200	3.89	3.04	2.65	2.41	2.26	2.14	1.98	1.80	1.69	1.62	1.52	1.45	1.42	1.32	1.19
400	3.86	3.02	2.62	2.39	2.23	2.12	1.96	1.78	1.67	1.60	1.49	1.42	1.38	1.28	1.13
∞	3.84	2.99	2.60	2.37	2.21	2.09	1.94	1.75	1.64	1.57	1.46	1.40	1.32	1.24	1.00

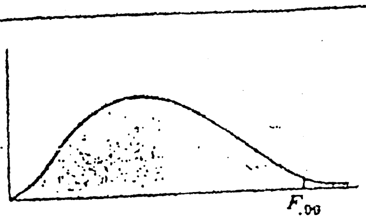
Source: G. W. Snedecor and W. G. Cochran, *Statistical Methods* (6th edition, 1967), Iowa State University Press, Ames, Iowa, by permission of the authors and publisher.

(5)

TABLE 51

99th PERCENTILE VALUES FOR THE F DISTRIBUTION

n_1 = degrees of freedom for numerator
n_2 = degrees of freedom for denominator
(shaded area = .99)



n_2 \ n_1	1	2	3	4	5	6	8	12	16	20	30	40	50	100	∞
1	4052	4999	5403	5625	5764	5859	5981	6106	6169	6208	6258	6286	6302	6334	6366
2	98.49	99.01	99.17	99.25	99.30	99.33	99.36	99.42	99.44	99.45	99.47	99.48	99.48	99.49	99.50
3	34.12	30.81	29.46	28.71	28.24	27.41	27.49	27.05	28.63	26.69	26.50	26.41	26.35	26.23	26.12
4	21.20	18.00	16.69	15.98	15.52	15.21	14.80	14.37	14.15	14.02	13.83	13.74	13.69	13.57	13.46
5	16.26	13.27	12.06	11.39	10.97	10.67	10.27	9.89	9.68	9.55	9.38	9.29	9.24	9.13	9.02
6	13.74	10.92	9.78	9.15	8.75	8.47	8.10	7.72	7.52	7.39	7.23	7.14	7.09	6.99	6.88
7	12.25	9.55	8.45	7.85	7.46	7.19	6.84	6.47	6.27	6.15	5.98	5.90	5.85	5.75	5.65
8	11.26	8.65	7.59	7.01	6.63	6.37	6.03	5.67	5.48	5.36	5.20	5.11	5.06	4.96	4.86
9	10.56	8.02	6.99	6.42	6.06	5.80	5.47	5.11	4.92	4.80	4.64	4.56	4.51	4.41	4.31
10	10.04	7.56	6.55	5.99	5.64	5.39	5.06	4.71	4.52	4.41	4.25	4.17	4.12	4.01	3.91
11	9.65	7.20	6.22	5.67	5.32	5.07	4.74	4.40	4.21	4.10	3.94	3.86	3.80	3.70	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.50	4.16	3.98	3.86	3.70	3.61	3.56	3.46	3.36
13	9.07	6.70	5.74	5.20	4.86	4.62	4.30	3.96	3.78	3.67	3.51	3.42	3.37	3.27	3.16
14	8.86	6.51	5.56	5.03	4.69	4.46	4.14	3.80	3.62	3.51	3.34	3.26	3.21	3.11	3.00
15	8.68	6.36	5.42	4.89	4.56	4.32	4.00	3.67	3.48	3.36	3.20	3.12	3.07	2.97	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	3.89	3.55	3.37	3.25	3.10	3.01	2.96	2.86	2.75
17	8.40	6.11	5.18	4.67	4.34	4.10	3.79	3.45	3.27	3.16	3.00	2.92	2.86	2.76	2.65
18	8.28	6.01	5.09	4.58	4.25	4.01	3.71	3.37	3.19	3.07	2.91	2.83	2.78	2.68	2.57
19	8.18	5.93	5.01	4.50	4.17	3.94	3.63	3.30	3.12	3.00	2.84	2.76	2.70	2.60	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.56	3.23	3.05	2.94	2.77	2.69	2.63	2.53	2.42
22	7.94	5.72	4.82	4.31	3.99	3.76	3.45	3.12	2.94	2.83	2.67	2.58	2.53	2.42	2.31
24	7.82	5.61	4.72	4.22	3.90	3.67	3.36	3.03	2.85	2.74	2.58	2.49	2.44	2.33	2.21
26	7.72	5.53	4.64	4.14	3.82	3.59	3.29	2.96	2.77	2.66	2.50	2.41	2.36	2.25	2.13
28	7.64	5.45	4.57	4.07	3.76	3.53	3.23	2.90	2.71	2.60	2.44	2.35	2.30	2.18	2.06
30	7.56	5.39	4.51	4.02	3.70	3.47	3.17	2.84	2.66	2.55	2.38	2.29	2.24	2.13	2.01
40	7.31	5.18	4.31	3.83	3.51	3.29	2.99	2.66	2.49	2.37	2.20	2.11	2.05	1.94	1.81
50	7.17	5.06	4.20	3.72	3.41	3.18	2.88	2.56	2.39	2.26	2.10	2.00	1.94	1.82	1.68
60	7.08	4.98	4.13	3.65	3.34	3.12	2.82	2.50	2.32	2.20	2.03	1.93	1.87	1.74	1.60
70	7.01	4.92	4.08	3.60	3.29	3.07	2.77	2.45	2.28	2.15	1.98	1.88	1.82	1.69	1.53
80	6.96	4.88	4.04	3.56	3.25	3.04	2.74	2.41	2.24	2.11	1.94	1.84	1.78	1.65	1.49
100	6.90	4.82	3.98	3.51	3.20	2.99	2.69	2.36	2.19	2.06	1.89	1.79	1.73	1.59	1.43
150	6.81	4.75	3.91	3.44	3.14	2.92	2.62	2.30	2.12	2.00	1.83	1.72	1.66	1.51	1.33
200	6.76	4.71	3.88	3.41	3.11	2.90	2.60	2.28	2.09	1.97	1.79	1.69	1.62	1.48	1.28
400	6.70	4.66	3.83	3.36	3.06	2.85	2.55	2.23	2.04	1.92	1.74	1.64	1.57	1.42	1.19
∞	6.64	4.60	3.78	3.32	3.02	2.80	2.51	2.18	1.99	1.87	1.69	1.59	1.52	1.36	1.00

Source: G. W. Snedecor and W. G. Cochran, *Statistical Methods* (6th edition, 1967), Iowa State University Press, Ames, Iowa, by permission of the authors and publisher.