THE OPEN UNIVERSITY OF SRI LANKA
BACHELOR OF TECHNOLOGY / BACHELOR OF SOFTWARE ENGINEERING – LEVEL 05
FINAL EXAMINATION – 2013/2014
MPZ5140/MPZ5160 – DISCRETE MATHEMATICS II
DURATION: THREE (03) HOURS

Date: 04th August 2014 **Time: 0930hrs – 1230hrs**

Instructions:

- **Answer six (06) questions only, selecting at least one (01) question from each section A, B and C.**
- **Number of pages in the paper – 06.**
- **All symbols are in standard notation.**

Section A

1. **(i)** Define a group in usual notation. [10%]

(ii) Determine the commutativity and associativity of each of the following binary operations.

(a) " $*$ " defined on $\mathbb{R}$ by $x * y = x + xy$ [25%]

(b) " $*$ " defined on $\mathbb{R}$ by $x * y = x(1 + y) + y(1 + x)$ [25%]

(iii) Prove that G is a group with respect to the binary operation defined on $\mathbb{R}$ by

$a * b = a + b + 2ab$ [25%]

(iv) Let G be a group and a and $b \in G$. Show that the equation $ax = b$ has unique solution in G. [15%]

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2. A binary operation " $*$ " on $\mathbb{Q}$ (rational numbers) is defined by

$a * b = a + b - ab$

(i) Find $(8 * 1/2)$ [20%]

(ii) Is $(\mathbb{Q}, *)$ a semi group? Is it commutative? [40%]

(iii) Find the identity element and the inverse element for " $*$ ". [40%]

3. (i) Define monoid in usual notation. [10%]

(ii) Determine whether each of the following set together with binary operation is a semi group monoid or not and in each case, if monoid then specify the identity element.

(a) $\mathbb{Z}^-$, where ordinary multiplication [20%]

(b) $\mathbb{Z}^+$, where " $*$ " is $a * b = a$ [20%]

(iii) Define a Homomorphism and Isomorphism for a group in usual notation. [20%]

(iv) Let T ={All even integers}. Show that the semi groups $(\mathbb{Z}, +)$ and $(T, +)$ are isomorphism. The mapping is given by $f: \mathbb{Z} \rightarrow T$. [30%]

Section B

4. (i) By drawing each of the following graphs, indicate which are simple graphs or not.

(a) $G_1 = \{V_1, E_1\}$ *where* $V_1 = \{1,2,3,4,5,6\}$ and

$E_1 = \{\{x, y\}, 2x + y \text{ is even and } x \leq y\}$ [20%]

(b) $G_2 = \{V_2, E_2\}$ *where* $V_2 = \{1,2,3,4,5,6,7\}$ and

$E_2 = \{\{1,2\}, \{1,3\}, \{1,4\}, \{2,5\}, \{2,6\}, \{3,6\}, \{4,5\}, \{4,6\}\}$ [15%]

(c) Determine the number of edges in a graph with 6 nodes, 2 of them degree 3 and 4 of degree 2. Draw two distinct such graphs. [20%]

(ii) Let G be the graph with $V(G) = \{1,2, \dots, 10\}$, such that two numbers "i" and "j" in $V(G)$ are adjacent if and only if $|i - j| \leq 3$. Draw the graph G and determine $E(G)$. [20%]

(iii) Three teams of three specialist soldiers each (a scout, a rifleman, a signaler) are to be sent into enemy territory. However some of the soldiers cannot work well with some others. The following table shows the soldiers their specializations and who they cannot work with.

Soldiers	Specialization	Can not cooperate with
1	scout	5,7
2	scout	5,6,7
3	scout	4,8
4	rifleman	3,7,9
5	rifleman	1,2
6	rifleman	2,8,9
7	signaler	1,2,4
8	signaler	3,6
9	signaler	4,6

Draw a multigraph to model the situation so that we may see how to form 3 man teams such that each specialization is represented and every member of the team can work with every other.

State clearly what the vertices represent and under what conditions two vertices are joined by an edge. [25%]

5. **(i)** Find the vertices n such that the complete graph has at least 1500 edges. [30%]

(ii) Define a "Tree" graph. [15%]

(iii) Draw all trees of four (4) vertices. [25%]

(iv) Draw a tree with 10 nodes each of which has either degree 1 or degree 3. Justify that it is impossible to draw such a tree with 11 nodes. [30%]

6. **(i)** Show that in any connected graph, there is even number of odd degree vertices. [20%]

(ii) G is the graph whose adjacency matrix A is given by $A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

(a) Without drawing a graph of G, determine whether G is connected or not. [20%]

(b) If $V(G) = \{a, b, c, d\}$ then find the number of paths of length four joining vertices d and b. [20%]

(c) Draw the graph of adjacency matrix A. [10%]

(iii) Let G be a graph of 8 vertices and 13 edges in which every vertex is of degree 3 or 4. How many vertices of degree 3 does G have? Construct one such graph G. [30%]

Section C

7. **(i)** Iterate the relationship $x_{n+1} - \lambda x_n(1 - x_n) = 0$, where $\lambda = 0.7$ and $x_0 = 0.4$ (at least 6 iteration steps are necessary) and draw the graph for the relationship. [30%]

(ii) Draw the graph for the relation $Z_{n+1} = {Z_n}^2$, where $Z_0 = 1.5 + 0.2\,i$. Find Z_5 and hence deduce Z_n as $n \to \infty$. [30%]

(iii) For the given relationship $x_{n+1} = \lambda x_n(1 - x_n)$, where $\lambda = 1.5$, obtain the convergent value for each of following case:

(a) $x_0 = 0.4$ [20%]

(b) $x_0 = 0.6$ [20%]

8. A three dimensional system is governed by following system of differential equations:

$$\frac{dx}{dt} - 3x - 2y + 2z = 0$$

$$\frac{dy}{dt} - 2x - 3y + 2z = 0$$

$$\frac{dz}{dt} + 2x + 2y - 3z = 0$$

At $t = 0$, $(x, y, z) = (1,0,1)$. Find the space value (x_t, y_t, z_t) for $t = 1,2$. [100%]

9. **(i)** What is a grammar? [10%]

(ii) Show that the string $(a + a) \ * \ \big((a - a) \ \div a\big)$ is a sentence governed by the grammar G, where $G = \big\{\{S, E\}, \{+, -, *, (\ \ ), \div\}, P, S\big\}$ and P is the set of productions. [20%]

$S \rightarrow SES$ $\qquad$ $S \rightarrow \div$

$S \rightarrow a$ $\qquad$ $S \rightarrow *$

$S \rightarrow +$ $\qquad$ $S \rightarrow (S)$

$S \rightarrow -$

(iii)(a) Let $L_1 = \{a, b, c\}$ *and* $L_2 = \{pq, r\}$. Find L_1L_2. [10%]

(b) If $L = \{01,10,110,100\}$, then find L^2. [10%]

(iv) Draw the directed graph that describes the DFA with the following state transition table. [20%]

States	Input	
	1	0
S_0	S_3	S_2
S_1	S_1	S_3
S_2	S_3	S_1
S_3	S_3	S_3

Initial state S_0 and accepting state S_3.

(v) Let M be a Mealy machine. Let $s \in S, a \in I$ and $x \in I^*$ and defined functions

$\delta : S \times I^* \rightarrow S$ and $\beta^* : S \times I^* \rightarrow O^*$ by

$\delta^*(s, \Omega) = s,$

$\delta^*(s, a.x') = \delta^*[\delta(s,a), x'],$

$\beta^*(s, \Omega) = \Omega,$

$\beta^*(s, a.x') = \beta(s,a)\beta^*[\delta(s,a), x'].$

The two frame binary pipeline devise hold up two binary digits as in the following table.

Contents	Input		Output	
	0	1	0	1
00	00	10	1	1
01	11	01	0	0
10	11	01	0	1
11	10	00	1	0

Find the two frame binary pipeline buffer and work out its response to the sequence 0101 from the state 01. [30%]

End