The Open University of Sri Lanka
Department of Electrical and Computer Engineering
ECX5241 – Distributed Parameter Systems
Final Examination – 2013/2014

Date: 2014-08-17 Time:0930-1230

The paper contains two sections A and B. Answer **four** questions by answering **anythreefrom section A** and **one question from section B**.

Section A

(20 Marks for each)

1.

(a) Explain what is a conservative vector field?

(b) Show that the divergence of the curl of any vector field is zero. That is $\nabla \cdot \nabla \times \boldsymbol{A} = 0$ for a vector field $\boldsymbol{A}$.

(c) Let $\underline{\boldsymbol{B}} = r^2 \underline{\boldsymbol{u}}_r + 2z\, \underline{\boldsymbol{u}}_z$. Determine the net flux of the vector field $\underline{\boldsymbol{B}}$ leaving a closed surface defined by $r = R$, $z = 0$, and $z = h$.

2.

(a) The heat flow vector $\boldsymbol{H} = k\nabla T$, where T is the temperature and k is the thermal conductivity. Show that $\nabla \cdot \boldsymbol{H} = 0$ where $= 50 \sin\frac{\pi x}{2} \cosh\frac{\pi y}{2}$.

(b) Let $V = (A \cos nx + B \sin nx)(Ce^{ny} + De^{-ny})$, where A,B, C and D are constants. Show that V satisfies Laplace's equation.

3.

(a) Write down the general form of a linear second-order PDE. What are the criterions for the parabolic, hyperbolic and elliptic equations? State a real time application/example for each equation.

(b) Explain why numerical methods are using for solving PDEs?

(c) Explain the Finite Element Method for solving PDEs?

4.

(a) Describe the "separation of variables method" in finding the solution for Laplace's equation in Cartesian coordinates.

(b) If $\nabla . \boldsymbol{E} = 0$, $\nabla . \boldsymbol{H} = 0$, $\nabla \times \boldsymbol{E} = -\frac{\partial H}{\partial t}$ and $\nabla \times \boldsymbol{H} = \frac{\partial E}{\partial t}$. Show that $\boldsymbol{E}$ and $\boldsymbol{H}$ satisfy the wave equation:

$$\nabla^2 u = \frac{\partial^2 u}{\partial t^2}$$

Section B

(40 Marks for each)

5.

A tightly stretched string with fixed ends at $x = 0$ and $x = l$ is initially in equilibrium position. It is set vibrating by giving to each of its points an initial velocity

$$\left(\frac{\partial y}{\partial t}\right)_{t=0} = b \sin\frac{\pi x}{l}$$

Find the displacement $y(x, t)$.

(Hint: $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$)

6.

Neglecting R and G find the emf $V(x, t)$ in a transmission line of length l, t seconds after the ends are suddenly grounded, if initially $I(x, 0) = I_0$ and $V(x, 0) = E \sin\frac{\pi x}{l}$.

(Hint: $\frac{\partial^2 V}{\partial x^2} = LC \frac{\partial^2 V}{\partial t^2}$).