THE OPEN UNIVERSITY OF SRI LANKA

DIPLOMA IN TECHNOLOGY– FOUNDATION LEVEL 02

FINAL EXAMINATION – 2008/2009

MPZ 2230 – MATHEMATICS – PAPER II

DURATION – THREE (03) HOURS

Registration No............................

DATE : 16th March 2009 **TIME: 9.30 a.m. – 12.30 p.m.**

ANSWER SIX (06) QUESTIONS ONLY.
YOU CAN USE NON PROGRAMMABLE CALCULATORS. YOU CAN'T USE MOBILE PHONES AS CALCULATORS.

01. The vertices Q and R of a triangle PQR lie on the line 3x-4y=0 and the y axis respectively. The side QR passes through (2/3, 2/3) and has slope λ;

    i. Find the coordinates of Q and R in terms of λ.

    ii. Show that $OQ = \left|\frac{10}{3}\frac{(1-\lambda)}{(3-4\lambda)}\right|$ and OR = $\left|\frac{2}{3}(1-\lambda)\right|$

    Where O is the origin.

    iii. If PQOR is a rhombus, find the two possible values of λ and the corresponding coordinates of P.

02. Find the condition that the circles $S_1 = x^2+y^2+2g_1x+2f_1\ y+c_1=0$ and $S_2 = x^2+y^2+2g_2x+2f_2y+c_2=0$ may touch and prove that, if they touch, the point of contact lies on each of the lines $2(g_1-g_2)x+2(f_1-f_2)y+c_1-c_2=0$ and $(f_1-f_2)x-(g_1, g_2)y+f_1g_2-f_2g_1=0$

    Show that the circles $S_1 \equiv x^2+y^2-2x+4y=0$ and $S_2 \equiv x^2+y^2-10x+20=0$ touch each other externally, and find the coordinates of A, the point of contact of the two circles.

    P is a point such that the length of the tangent from P to the first circle is λ (a constant) times that of the tangent from P to the second. Prove that if $\lambda^2 \neq 1$ the locus of P is a circle through A, and find its equation in terms of λ.

03. Find the equation of the normal to the parabola $y^2 = 4ax$ at the point P $(at^2,2at)$. The coordinate of the point Q at which the normal at P meets the parabola again is $(aT^2,2aT)$. Show that $t^2+tT+2=0$ and deduce that $T^2 \geq 8$.

    The line 3y = 2x + 4a meets the parabola at the point H and K show that the normals at the points H and K meet on the parabola.

04. $P_1 \equiv (a\cos\alpha, b\sin\alpha)$ and $P_2 \equiv (a\sin\alpha, b\cos\alpha)$ be two distinct points on the ellipse $S \equiv \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$

Find the equation of the chord P_1P_2. Write down the equation of the tangent to the ellipse S = 0 at the point ($a\cos\theta$, $b\sin\theta$).

Let T be the point of intersection of the tangents to S = 0 at the points P_1 and P_2. Find the coordinates of T.

Deduce that locus of the point T is by –ax = 0.

05. Show that the equation of the tangent to the rectangular hyperbola $xy = c^2$ at the point P $\equiv$(cp,c/p) is $x + p^2 y = 2cp$.

The tangent at P $\equiv$(cp,c/p) to the hyperbola $xy = c^2$ meets the x axis at the point Q and the line through Q parallel to the y axis meets the hyperbola at the point R. The line through P parallel to the x axis meets the y axis at the point S.

Show that the line RS is the tangent to the hyperbola at R.

The tangents to the hyperbola at P and R intersect at a point T. Find the coordinates of the point T. Show that the locus of T, as P varies on the hyperbola, is also a rectangular hyperbola.

06. a) $f(\theta) \equiv \frac{1}{5\cos\theta + 3\cos(\theta + \pi/3) + 8}$

i. Show that $\frac{1}{15} \le f(\theta) \le 1$

ii. Solve the equation $9f(\theta) = 2$

b) Show that $Sec^2(Tan^{-1}2) + Cosec^2(Tan^{-1}3) = \frac{55}{9}$

c) Solve the equation $Tan^{-1}(x+1) + Tan^{-1}(x-1) = Tan^{-1}\left(\frac{8}{31}\right)$ for $|x| < \sqrt{2}$.

07. (a) In a triangle, the largest angle twice the size of the smallest angle and the longest side is 1 ½ times the length of the shortest side. Show that smallest angle of the triangle is $Cos^{-1}(3/4)$. Given that the length of the remaining side is 10cm, find the lengths of the other two sides.

(b) By using the sine Rule for a triangle, prove in the usual notation for a triangle ABC that $(a+b+c)\left[Tan\frac{A}{2} + Tan\frac{B}{2}\right] = 2c\cot\frac{C}{2}$. Hence or otherwise, show that $\frac{a+b-c}{a+b+c} = Tan\frac{A}{2}.Tan\frac{B}{2}$.

08. i. The complex number Z_0 is given by $Z_0 = \frac{3+j}{2-j}$

a) Express Z_0 in the form a+bj where a, b $\in \mathbb{R}$;

b) Find the modulus and argument of Z_0.

c) Sketch an argond diagram showing the point representing the complex number Z_0 . Show on the same diagram the locus of the point representing the complex number Z such that $|Z - Z_0| = 1$.

Using your diagram calculate the least value of $|Z|$ for points on this locus.

ii. The equation $2x^3+x^2+25 = 0$ has one real root and two complex roots.

a) Verify that 1+2j is one of the complex roots.

b) Write down the other complex root of the equation and find the real root of the equation.

c) Sketch an Argond Diagram showing the point representing the complex number 1+2j. Indicate on the same diagram the laws of the points representing the complex number Z. Satisfying the relation $|Z| = |Z-1-2j|$.

09. State De Moivere's Theorem for a positive integral index.

i. Using the above theorem, show that $(1+j\tan\theta)^n + (1-j\tan\theta)^n = \frac{2\cos n\theta}{\cos^n\theta}$

Where n $\in Z^+$ and $Cos\theta \neq 0$

Using this result, show that the roots of the equation $(1+z)^4+(1-z)^4 = 0$ are $\pm jTan\frac{\pi}{8}$, $\pm jTan\frac{3\pi}{8}$,

Hence show that

$Tan^2\frac{\pi}{8} = 3 - 2\sqrt{2}$

ii. Evaluate $(1-\sqrt{3}j)^4(1+j)^3$

10. a) Three friends Amal, Bnadu and Chamil agree to meet at the theatre. Amal cannot remember whether they agree to meet at the Regal or the Savoy and toss a coin decide which theatre to go to. Bandu also tosses a coin to decide between the Savoy and the Majestic city Chamil tosses a coin to decide whether to go to the Regal or not and in this latter case he tosses again to decide between the Savoy and the Majestic City . Draw a complete tree diagram.

Find the probability that

i. Amal and Bandu meet
ii. Bandu and Chamil meet
iii. Amal, Bandu and Chamil all meet.
iv. Amal, Bandu and Chamil all go to different places
v. At least two meet.

b) Three events X, Y and Z are defined in the sample space. The events X and Z are mutually exclusive. The events X and Y are independent. Given that $P(X) = \frac{1}{3}$, $P(Z) = \frac{1}{5}$, $P(X \cup Y) = \frac{2}{3}$

Find $P(X \cup Z), P(Y), P(X \cap Y)$

Given also that $P(Y \cup Z) = \frac{3}{5}$ determine whether or not events Y and Z are independent.

11. A tyre manufacturer conducts on a particular types of tyre. A sample of 100 tyres is put on test and the distance travelled by the tyres before reaching the legal limit of tyre wear are shown in Table.

Distance/1000km	No. of tyres.
0 – 15	6
15 – 25	16
25 – 35	34
35 – 45	26
45 – 55	14
55 – 65	4

Calculate (i) mode of the distance.
(ii) medium of the distance.
(iii) mean of the distance.
(iv) standard deviation of the distance.

12. a) A particle A of mass m is held on the surface of a fixed smooth solid sphere centre O and radius a at point P such that OP makes an acute angle $\cos^{-1}$ (¾) with upward vertical, and then released. Prove that, when OA makes an angle θ with the upward vertical, the velocity υ of the particle is given by $\upsilon^2 = \frac{1}{2}ga(3-4\cos\theta)$.

Provided that the particle remains on the surface of the sphere, find the normal reaction on the particle at this time. Deduce that the particle leaves the surface when OA makes an angle π/3 with the upward vertical.

b) A particle of mass 2kg is attached to one end of an elastic string of natural length 1m whose other end is fixed to a point A on a smooth horizontal plane. The particle is pulled across the plane to a point C where AC = 1.5m and released from rest at C. B is a point on AC such that AB = 1m. If the modulus of elasticity of the string is 10N.

Show that

a) from C to B the particle performs Simple Harmonic Motion with centre B.

b) The time taken to travel from C to B is $\frac{\sqrt{5}\pi}{10}s$.

c) The speed at B is $\frac{\sqrt{5}}{2}ms^{-1}$

d) The particle then travel for $\frac{4}{5}\sqrt{5}\ s$ with constant speed.