#### THE OPENUNIVERSITY OF SRI LANKA

Department of Civil Engineering

Bachelor of Technology - Level 5

CEX 5231 - MECHANICS OF FLUIDS

FINAL EXAMINATION - 2014/2015

Time Allowed: Three Hours

Date: 07th September, 2015



Time: 9:30-12:30 hrs

# ANSWER **ALL THREE** QUESTIONS IN PART A AND **ANY TWO** QUESTIONS IN PART B. ALL QUESTIONS CARRY EQUAL MARKS.

#### PART A

Answer all three questions in this section. Graph papers are available on request.

1)

- a) What is a boundary layer?
- b) Describe the phenomenon of boundary layer separation when flow takes place over a curved surface.
- c) What are the causes of separation of boundary layers and how can separation be controlled?

A uniform, horizontal flow approaches a horizontal thin flat plate as shown in Figure 1 The top surface of the plate is smooth while the bottom surface is rough.

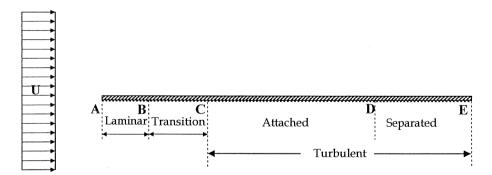


Figure 1

- d) Sketch the velocity profiles over the top surface of the plate considering the given regions. Identify the profiles.
- e) Compare the thickness of the boundary layers on the top and bottom surfaces of the plate. Explain your answer.

- a) What is meant by Specific Energy in an open channel flow?
- b) Explain, using a neat diagram, the relationship between the specific energy of an open channel flow and the total head at any point in the flow. State all your assumptions.
- c) An open channel has a uniform rectangular cross-section that is 0.5 m in width, and carries a discharge of 17 m<sup>3</sup>s<sup>-1</sup>. A free hydraulic jump forms at some point of the flow.
  - i) Derive a relationship between the specific energy and the depth of flow for the above channel.
  - ii) Determine the conjugate depth corresponding to the depth of 0.7 m before the jump.

Plot on a graph paper the specific energy curve for depths in the range 0.5 m to 3.0 m using the same scale for depth and specific energy.

Using the graph determine;

- iii) the critical depth of the flow.
- iv) the minimum specific energy.
- v) alternate depths for specific energy of 2.3 Nm/N.
- vi) the loss of energy in the jump.

(The equation 
$$\frac{y_2}{y_1} = \frac{1}{2} \left( \sqrt{1 + 8F_{r_1}^2} - 1 \right)$$
 can be useful)

3) A fluid flows between two flat plates, as shown in Figure 3. The lower plate is at an angle  $\theta$ - $\delta\theta$  to the horizontal while the upper plate is at an angle  $\theta$ + $\delta\theta$  to the horizontal.

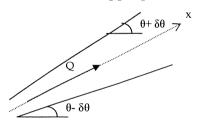
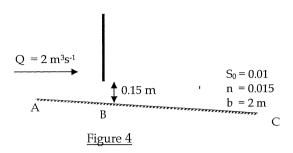


Figure 3

- a) Apply the principles of conservation of mass and conservation of momentum to an appropriate control volume and derive differential equations governing the variation of the average velocity,  $\mathbf{v}(\mathbf{x})$  and pressure  $\mathbf{p}(\mathbf{x})$  along the flow direction. Your derivation should include the effects of friction between the fluid and the plate. State any assumptions that you make.
- b) What is the relationship between the equations that you derived in section a) and the Bernoulli equation? Explain your answer.
- c) Discuss whether the result obtained in section a) will remain valid as  $\delta\theta$  increases. Explain your answer.

## PART B Answer any two questions in this section.

4)



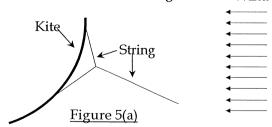
A long open channel ABC has a width of 2 m, a slope of 0.01 and a Manning's coefficient of 0.015. The channel carries a steady discharge of  $2 \text{ m}^3/\text{s}$ . A gate is placed in this channel at B as shown in Figure 4. The opening of the gate is 0.15 m and uniform flow is observed far upstream and downstream of the gate.

- a) Calculate the critical depth for the given channel and discharge.
- b) Calculate the uniform depth for the given channel and discharge.
- c) Calculate the depth of flow just upstream of the gate at B.
- d) Show that there is a free hydraulic jump somewhere near the gate.
- e) Sketch the profile of the flow (variation of flow depth) from the uniform flow far upstream of the gate to the uniform flow far downstream of the gate. Identify the flow profile type (from M1, M2, M3, S1, S2, S3) of each section and indicate the location of any hydraulic jumps.
- f) Explain how you would find the exact location of the jump.

Note: The equations 
$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - F_r^2}$$
 and  $\frac{y_1}{y_2} = \frac{1}{2} \left( \sqrt{1 + 8F_{r2}^2} - 1 \right)$  may be used.

5)

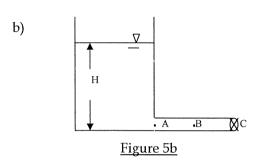
a) The side view of a two- dimensional kite is shown in figure 5(a). The kite is flying in a steady wind that blows from right to left, as shown in the figure. The kite is held in place by a string, which is also shown in the figure.



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### (Question 5 continued...)

- i) Define the terms "Drag Force" and "Lift Force" as applied to a body held stationary in a flowing fluid.
- ii) Using a sketch of the stream lines of the wind flow past a kite, explain how flow separation can result in Drag and Lift Forces on the kite.
- iii) Mark all the forces acting on the kite on a neat diagram.
- iv) Write down the equations representing the balance of these forces when the kite is flying in a stable position.
- v) Explain why pulling the string will cause a kite to rise when it is flying in a uniform wind.
- vi) Once the kite reaches a certain level running will not be necessary. Explain why.



Water flows steadily out of a large tank, through a horizontal pipeline ABC as shown in Figure 5b. The water level in the tank is H, as shown in the figure. There is a valve at C. It is observed that a sudden closure of the valve at C results in a rapid increase of the pressure just upstream of the valve.

- i) Explain why the sudden closure of the valve results in a rapid increase of the pressure just upstream of the valve.
- ii) Sketch the variation of the pressure at the locations A, B and C in the pipeline with time after the sudden closure of the valve on the same graph. Explain your answer.

6)

- a) State the Buckingham's pi theorem.
- b) What do you mean by repeating variables? How are the repeating variables selected for dimensional analysis?
- c) Define the terms; model, prototype, model analysis and hydraulic similitude.
- d) Explain the different types of hydraulic similarities that must exist between a prototype and its model. Are these similarities truly attainable? If not, why?

The power P developed by a water turbine depends on the rotational speed N, operating head H, gravity g, diameter D and breadth B of the runner, density  $\rho$  and viscosity  $\mu$  of water.

(Question 6 continued...)

e) Show by dimensional analysis that the relationship between P and the other variables can be expressed as

$$P = \rho D^5 N^3 f \left[ \frac{H}{D}, \frac{D}{B}, \frac{\rho D^2 N}{\mu}, \frac{ND}{\sqrt{gH}} \right]$$

Two reservoirs, A and B, are separated by a mountain. as shown in Figure 7. The water surface level of reservoir B is 50 m lower than that of reservoir A. It is decided to pump water from A to B using a pipeline XYZ that passes over the mountain as shown in figure 7. Two identical pumps are used in parallel at X as shown in figure 7. A turbine is located at Z to recover some of the energy. The total length of the pipeline is 2 km while the pipe has a diameter of 10 cm and a friction factor of 0.015.

One of the pumps was tested at its operating speed and the results are given in Table 7a. The turbine was also tested at its operating speed and the results are given in Table 7b.

- a) Calculate the discharge in the pipe line when the pumps and turbine are run at the respective operating speeds. State all your assumptions and explain your answer.
- b) What proportion of energy supply to the pumps is recovered by the turbine?

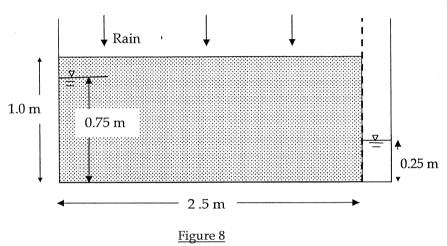
Discharge (l/s)	0	10	20	30	40
Head (m)	100	85	70	50	25
Efficiency	-	44	<i>7</i> 5	80	40
<u>Table 7a</u>					
Discharge (l/s)	0	10	20	30	40
Head (m)	40	35	28	20	10
Efficiency	_	50	70	85	60
Table 7b					

Y 2km 50 Z B

Figure 7

8)

The cross-section of a sand box of length 2.5 m and width 0.75 m, used in a laboratory experiment is shown in Figure 8. The box is filled with sand of a uniform grain size and packing density which has a permeability of 2 m/day to a depth of 1 m. Rain is simulated to fall on to the surface of the sand box uniformly at a steady rate. The wall on the left hand side is made impervious while the wall on the right hand side is pervious, as shown in the figure. A constant water level of 0.25 m above the bottom of the sand layer is maintained on the other side of the pervious wall on the right hand side by allowing the water to flow out. The rain infiltrates the sand and a steady water level of 0.75 m is set up inside the sand layer.



- a) Derive, from first principles, a differential equation governing the height of the free surface above the bottom of the sand layer. State all your assumptions.
- b) State the boundary conditions you would use to solve the governing equation and explain their physical significance.
- c) Solve the differential equation derived in a) and obtain an equation for the height of the free surface above the bottom of the sand layer.
- d) Sketch the height of the free surface in the sand layer.
- e) Calculate the rate of rainfall.

It was decided to allow water to flow out from both walls of the box while keeping the rainfall the same. After conditions become steady it was observed that 20% of the inflow from the rainfall flows out of the left side of the tank.

f) Sketch the shape of the free surface in the sand layer incorporating new conditions. Compare and explain the differences in the sketches obtained in d) and f).