00140

THE OPEN UNIVERSITY OF SRI LANKA

Bachelor of Technology (Civil) – Level 5

CEX5233 STRUCTURAL ANALYSIS

FINAL EXAMINATION - 2014/2015

Time Allowed – 3 Hours

Date: 28th August 2015 Time: 09.30 – 12.30 Hrs

This paper consists of **Eight (8)** questions. Answer **any Five (5)** questions.

All questions carry equal marks

QUESTION 1

(i) State **St. Venant's Principle** and briefly explain its importance in structural analysis. (3 Marks)

(ii) A simply supported beam of length 'l' is subjected to a uniformly distributed load 'w' as shown in Figure Q1. The beam has a rectangular cross section of breath 'b' and depth 'd'.

(a) Show that the function given below is a possible stress function. (5 Marks)

$$\emptyset = \frac{A}{6}\left(l^2y^3 - x^2y^3 + \frac{1}{5}y^5\right) + Bx^2 + Cx^2y$$

(b) Determine constants A, B and C applying the relevant boundary conditions. (6 Marks)

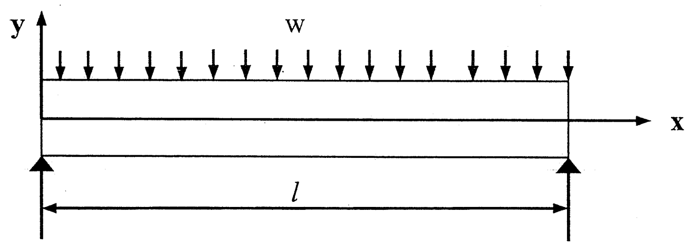

Figure Q1: Simply supported beam

(iii) Briefly describe the following experimental stress measurement techniques.

(a) Strain gauge method (2 Marks)

(b) Grid method (2 Marks)

(c) Photoelasticity method (2 Marks)

QUESTION 2

(i) List the three conditions that a structure must be satisfied on the point of collapse. (3 Marks)

(ii) Consider a beam fixed at both ends as shown in Figure Q2(a). Flexural rigidity of the beam is EI.

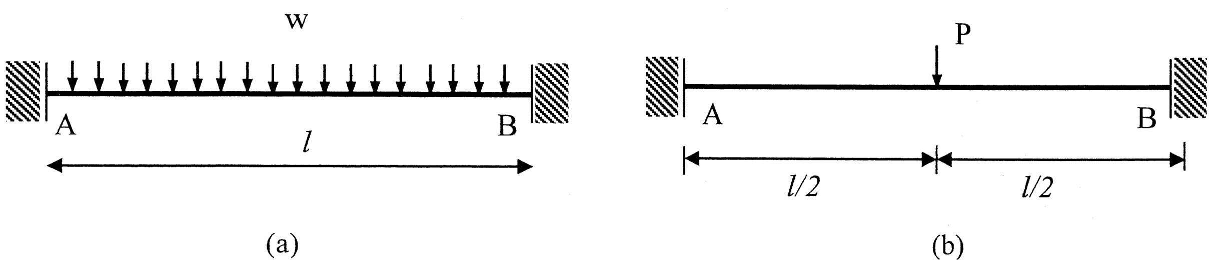

Figure Q2: Fixed supported beam with: (a) a uniformly distributed load; (b) central point load

a) Determine the ratio of plastic moment (M_p) to yield moment (M_y) for a circular beam with diameter "*d*". (6 Marks)

b) Determine the collapse load (w_{ult}) of the beam shown in Figure Q2(a) using the virtual work approach. You can consider that plastic moment of the beam as M_P. (4 Marks)

c) If the uniformly distributed load is removed and a point load (P) is applied as shown in Figure Q2(b), determine the collapse load (P_{ult}) using the virtual work approach. (4 Marks)

d) Obtain the ratio of $\frac{w_{ult}l}{P_{ult}}$ and discuss the practical significance of the result. (3 Marks)

QUESTION 3

(i) General equilibrium equation is written as $\frac{\partial \sigma_{ij}}{\partial x_j} + F_i = 0$ in usual notations. Using this, write down equilibrium equations in three orthogonal directions. (3 Marks)

(ii) The stress tensor at a point of a particular stress field is given below.

$$\sigma_{ij} = \begin{bmatrix} 3000 & 1000 & 1000 \\ 1000 & 0 & 2000 \\ 1000 & 2000 & 0 \end{bmatrix} N/m^2$$

a) Determine six independent stress components. (4 Marks)

b) Determine the stress invariants. (4 Marks)

c) Determine the principal stresses. (4 Marks)

d) Determine the directions of principal axes. (5 Marks)

QUESTION 4

(i) Explain "**Degree of Statical Indeterminacy**" of a structure using a neat sketch. (4 Marks)

(ii) Consider the continuous beam shown in Figure Q4. Flexural rigidity of members is equal to EI. Uniformly distributed load (w) is acting on member AB and a concentrated load (wl) is acting on the member CD at E.

a) Determine the degree of statical indeterminacy of the beam. (3 Marks)

b) Draw a released structure. (3 Marks)

c) Determine the flexibility matrix for the drawn released structure. (4 Marks)

d) Determine bending moments at B and C. (6 Marks)

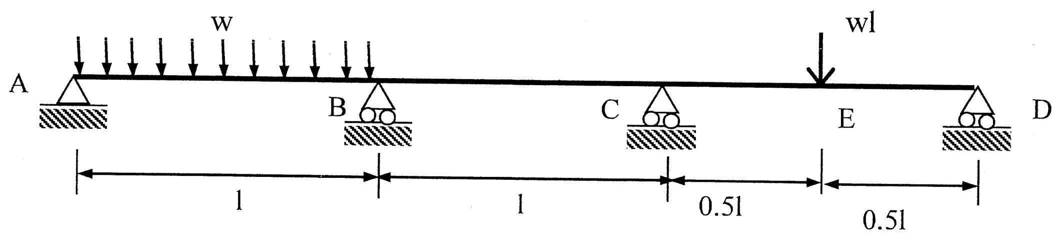

Figure Q4: A continuous beam with a uniformly distributed load and a point load

QUESTION 5

(i) Explain "**Kinematic Indeterminacy**" of a structure using a neat sketch. (4 Marks)

(ii) Consider the frame structure shown in Figure Q5. Flexural rigidities of members AB and BC are the same. Find the free nodal displacements at B using the displacement method. You can neglect the axial deformation. (10 Marks)

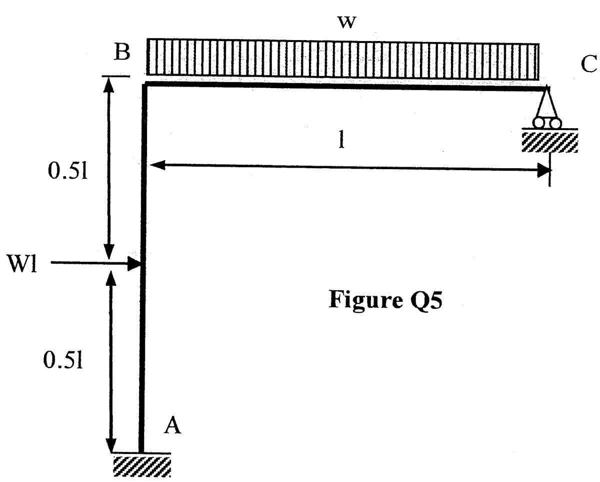

Figure Q5

(iii) Using results found in (ii), determine the bending moment at A. (6 Marks)

QUESTION 6

(i) Describe conceptual difference between elastic design and plastic design of beams and frames. (3 Marks)

(ii) A two-bay frame structure is shown in Figure Q6. Dimensions and plastic moments of the beam are given in the figure.

(a) Draw failure mechanisms. (3 Marks)

(b) Determine load factors for each failure mechanism. (7 Marks)

(c) Determine the most probable failure mechanism. (5 Marks)

(d) Explain how you can ensure the unique solution. (2 Marks)

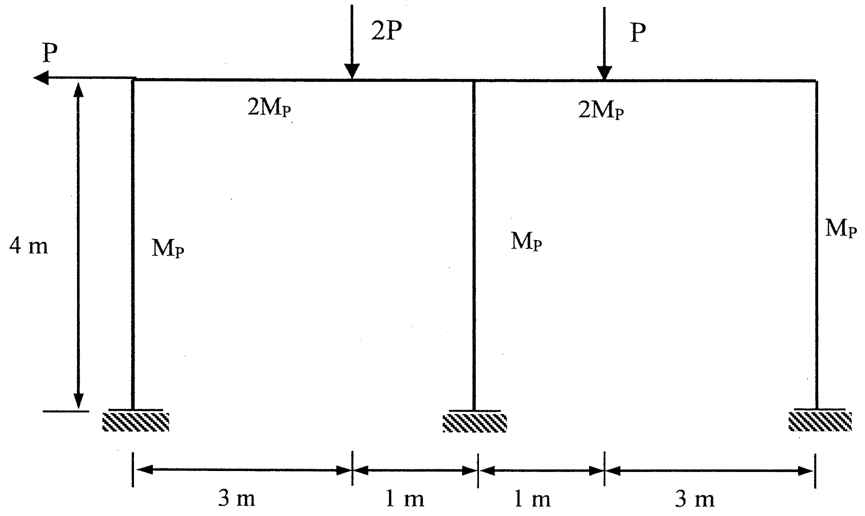

Figure Q6: Two-bay frame structure

QUESTION 7

(i) A rectangular plate of **sides *a* and *b*** is simply supported on the four sides and subjected to a distributed load q = q(x,y) as shown Figure Q7. Show that the lateral deflection of the plate is given by

$$w = \frac{1}{\pi^4 D} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{a_{mn}}{\left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^2} Sin \frac{m\pi x}{a} Sin \frac{n\pi y}{b} dxdy$$

where $a_{mn} = \frac{4}{ab} \int_0^a \int_0^b q.Sin \frac{m\pi x}{a} Sin \frac{n\pi y}{b} dxdy$ (14 Marks)

Hint: Express q as a double trigonometric series given by

$$q = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} Sin \frac{m\pi x}{a} Sin \frac{n\pi y}{b}$$

(ii) Using above results, show that the maximum deflection of a **simply supported square plate** of sides "**a x a**" ,loaded with a uniformly distributed load q_o is given by

$$w_{max} = \frac{4q_0 a^4}{\pi^6 D}$$

(6 Marks)

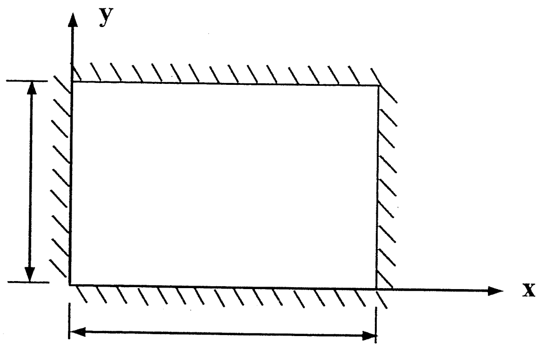

Figure Q7

QUESTION 8

(i) Briefly describe the following definitions used in the membrane theory of thin shells.

(a) Middle surface

(b) Normal plane

(c) Gaussian curvature

(4 Marks)

(ii) A spherical dome with radius 'a' is supported as shown in Figure Q8 and carries a uniformly distributed load 'w' per unit plan area. The shell thickness is 'h'.

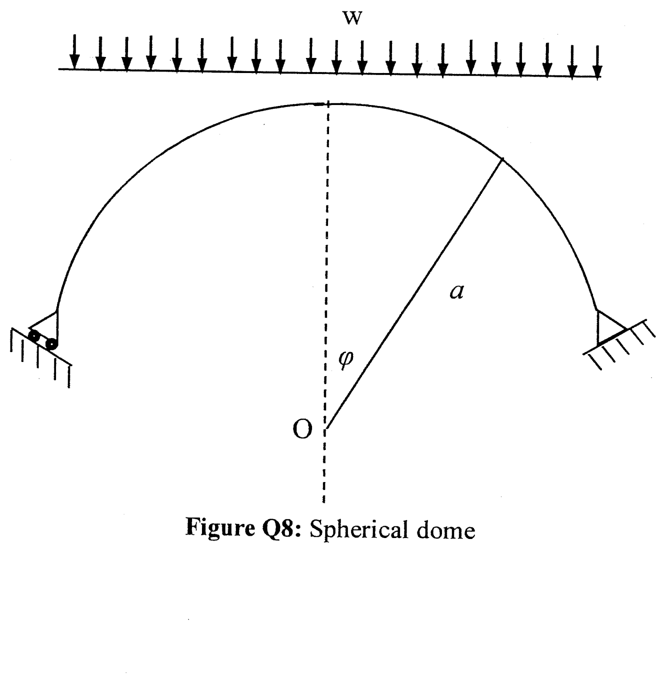

Figure Q8: Spherical dome

(a) Derive expressions for meridinal and hoop stresses in the spherical dome. (10 Marks)

(b) Show that the practical limitation of angle φ is 45^0, to prevent the development of tensile stresses in the dome.

(4 Marks)

(c) If angle φ is to be increased more than 45^0, describe alterations that you would propose in the design of concrete dome. (2 Marks)

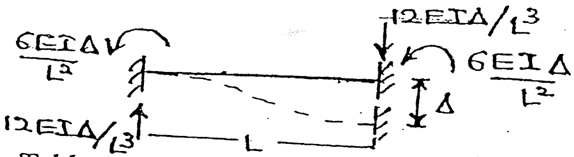

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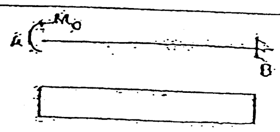
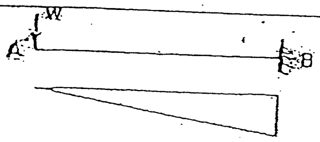
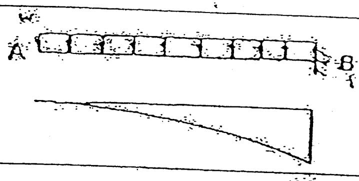
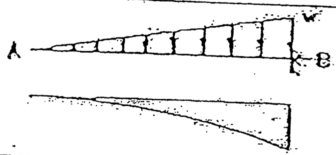
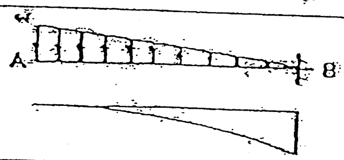
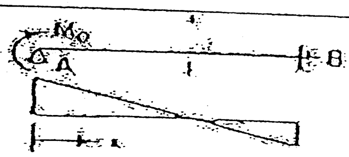
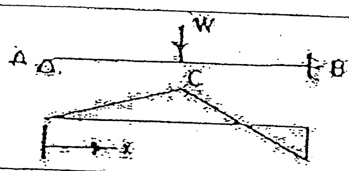
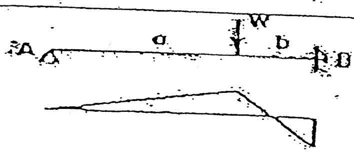
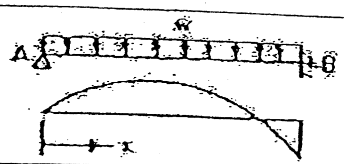
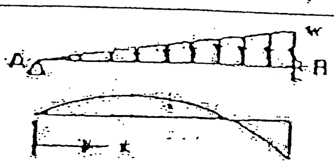
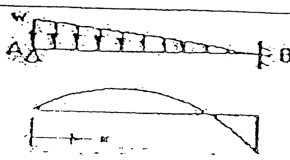
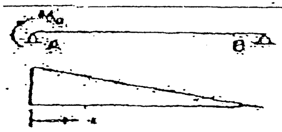
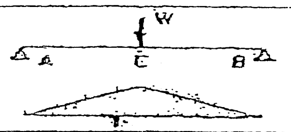
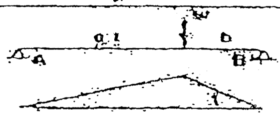
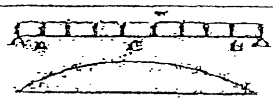
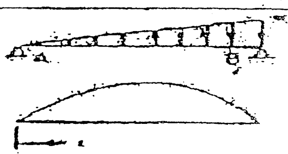
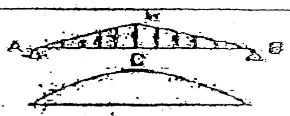
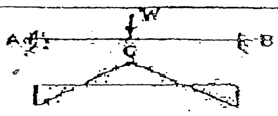
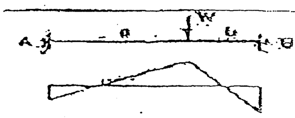
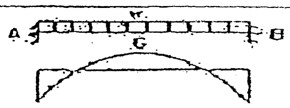
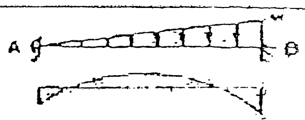
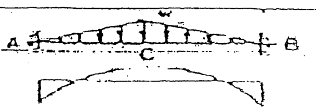
Table 1

Formulas for Beams

Structure	Shear	Moment	Slope	Deflection ↓
Cantilever Beam				
	0	M_o	$\theta_A=\frac{M_oL}{EI}$	$Y_A=\frac{M_oL^2}{2EI}$
	W	$M_B=-WL$	$\theta_A=-\frac{WL^2}{2EI}$	$Y_A=\frac{WL^3}{3EI}$
	$S_B=-WL$	$M_B=-\frac{WL^2}{2}$	$\theta_A=-\frac{WL^3}{6EI}$	$Y_A=\frac{WL^4}{8EI}$
	$S_B=-\frac{WL}{2}$	$M_B=-\frac{WL^2}{6}$	$\theta_A=-\frac{WL^3}{24EI}$	$Y_A=\frac{WL^4}{8EI}$
	$S_B=-\frac{WL}{2}$	$M_B=-\frac{WL^2}{2}$	$\theta_A=-\frac{WL^3}{8EI}$	$Y_A=\frac{11WL^4}{120EI}$
Propped Cantilever				
	$S_A=-\frac{3M_o}{2L}$	$M_B=-\frac{M_o}{2}$	$\theta_A=-\frac{M_oL}{4EI}$	$Y_{max}=\frac{M_oL^2}{27EI}$ at $x=\frac{L}{3}$
	$S_A=-\frac{3M_o}{2L}$	$M_B=-\frac{3WL}{16}$ $M_c=\frac{5WL}{32}$	$\theta_A=\frac{WL^2}{32EI}$	$Y_{max}=0.00962\frac{WL^3}{ELI}$ at $x=0.4471$
	$S_A=\frac{Wb^2}{2L^3}(a+2L)$ $S_B=-\frac{Wa}{2L^3}(3L^2-a^2)$	$M_B=-\frac{Wab}{L^2}(a+\frac{b}{2})$	$\theta_A=\frac{Wab^2}{4EIL}$	$Yo=\frac{Wa^2b^2}{12EIL^3}(3L+a)$
	$S_A=+\frac{3WL}{8}$	$M_B=-\frac{WL^2}{8}$	$\theta_A=\frac{WL^3}{48EI}$	$Y_{max}=0.0054\frac{WL^4}{ELI}$ at $x=0.422L$
	$S_A=+\frac{WL}{10}$	$M_{max}=0.03WL^2$ at $x=0.447L$ $M_B=-\frac{WL^2}{15}$	$\theta_A=\frac{WL^3}{120EI}$	$Y_{max}=0.00239\frac{WL^4}{EI}$ at $x=0.447L$
	$S_A=+\frac{11WL}{40}$	$M_{max}=0.0423WL^2$ at $x=0.329L$ $M_B=-\frac{7WL^2}{120}$	$\theta_A=\frac{WL^3}{80EI}$	$Y_{max}=0.00305\frac{WL^4}{EI}$ at $x=0.402L$

Annex

Table 1 Formulas for Beams

Structure	Shear	Moment	Slope	Deflection ↓
Simply supported Beam				
	$S_A = -\frac{M_o}{L}$	M_o	$\theta_A = \frac{M_o L}{3EI}$ $\theta_B = -\frac{M_o L}{6EI}$	$Y_{max} = 0.062 \frac{M_o L^2}{EI}$ $at\ x = 0.422L$
	$S_A = \frac{W}{2}$	$M_o = \frac{WL}{4}$	$\theta_A = -\theta_B = \frac{WL^2}{16EI}$	$Y_c = \frac{WL^3}{48EI}$
	$S_A = \frac{Wb}{L}$ $S_B = \frac{Wa}{L}$	$M_o = \frac{Wab}{L}$	$\theta_A = \frac{Wab}{6EIL}(L+b)$ $\theta_B = -\frac{Wab}{6EIL}(L+a)$	$Y_o = \frac{Wa^2b^2}{3EIL}$
	$S_A = \frac{WL}{2}$	$M_c = \frac{WL^2}{8}$	$\theta_A = -\theta_B = \frac{WL^3}{24EI}$	$Y_c = \frac{5WL^4}{384EI}$
	$S_A = \frac{WL}{6}$ $S_B = \frac{WL}{3}$	$M_{max} = 0.064WL^2$ $at\ x = 0.577L$	$\theta_A = \frac{7WL^3}{360EI}$ $\theta_B = -\frac{8WL^3}{360EI}$	$Y_{max} = 0.00652 \frac{WL^4}{EI}$ $at\ x = 0.519L$
	$S_A = \frac{WL}{4}$	$M_c = \frac{WL^2}{12}$	$\theta_A = -\theta_B = \frac{5WL^3}{192EI}$	$Y_c = \frac{WL^4}{120EI}$
Fixed Beams				
	$S_A = \frac{W}{2}$	$M_c = \frac{WL}{8}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{WL^3}{192EI}$
	$S_A = \frac{Wb^2}{L^3}(3a+b)$ $S_B = \frac{Wa^2}{L^3}(3b+a)$	$M_A = -\frac{Wab^2}{L^2}$ $M_B = -\frac{Wba^2}{L^2}$	$\theta_A = \theta_B = 0$	$Y_o = \frac{Wa^3b^3}{3EIL^3}$
	$S_A = \frac{WL}{2}$	$M_A = M_B = -\frac{WL^2}{12}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{WL^4}{384EI}$
	$S_A = \frac{3WL}{20}$ $S_B = \frac{7WL}{20}$	$M_A = -\frac{WL^2}{30}$ $M_B = \frac{WL^2}{20}$	$\theta_A = \theta_B = 0$	$Y_{max} = 0.00131 \frac{WL^4}{EI}$ $at\ x = 0.525L$
	$S_A = \frac{WL}{4}$	$M_A = M_B = -\frac{5WL^2}{96}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{0.7WL^4}{384EI}$