THE OPEN UNIVERSITY OF SRI LANKA
FACULTY OF ENGINEERING TECHNOLOGY
POSTGRADUATE DIPLOMA IN TECHNOLOGY IN INDUSTRIAL ENGINEERING
FINAL EXAMINATION - 2006/2007
MEX7211/MEP1201 - OPERATIONS RESEARCH
DATE : 26 MARCH 2007 031
TIME : 0930-1230 HRS
TIME DURATION : *THREE (03) HOURS*

INSTRUCTIONS:

(a) Answer any five (05) questions only. All questions carry equal marks.

(b) Normal distribution and P_o values for multiple-server model tables are attached to the question paper.

QUESTION 1.

In each of the following three situations, use the binomial, Poisson or normal distribution according to which is the most appropriate. In each case, explain why you selected the distribution and draw attention to any features which supports or casts doubt on the choice of distribution.

1.1 Situation 1

The lifetimes of a certain type of electrical components are distributed with a mean of 800 hours and a standard deviation of 160 hours.

1.1.1 If the manufacturer replaces all components that fall before the guaranteed minimum lifetime of 600 hours, what percentages of the components have to be replaced?

1.1.2 If the manufacturer wishes to replace only the 1% of components that have the shortest life, what value should be used as the guaranteed lifetime?

1.1.3 What is the probability that the mean lifetime of a sample of 25 of these electrical components exceeds 850 hours?

(10 marks)

1.2 Situation 2

A greengrocer buys peaches in large consignments directly from a wholesaler. In view of the perishable nature of the commodity, the greengrocer accepts that 15% of the supplied peaches will usually not be saleable. As he cannot check all of the peaches individually, he selects a single batch of 10 peaches on which to base his decision of whether to purchase a large consignment or not. If no more than two of these peaches are unsatisfactory, the greengrocer purchases the consignment.

Determine the probability that, under normal supply conditions, the consignment is purchased. *(05 marks)*

1.3 **Situation 3**

Vehicles pass a certain point on a busy single carriageway road at an average rate of two per ten second interval.

Determine the probability that more than three cars pass this point during a twenty-second interval.

(05 marks)

QUESTION 2

2.1 What is hypothesis testing?

2.2 Define Type I and Type II errors. *(03 marks)*

2.3 In a dispute between a particular company and a trade union, the union claimed that the mean number of hours overtime worked by maintenance men each week was 5. The Company disagree and so to test the claim, a sample of 200 men were chosen at random with the following results: *(04 marks)*

Number of Hours Overtime	Number of Men
0 - 2	27
2 - 4	91
4 - 6	42
6 - 8	23
8 - 10	10
10 - 12	5
12 - 14	2

Do you accept the claim? Briefly explain your answer.

(13 marks)

QUESTION 3

The Crown Chemical Company manufactures three products, Rostic, Senopure and Taltrate by mixing together four raw materials, Avex, Bumax, Ceedex and Dorax. The table $Q_3(a)$ gives information concerning the constituents required to produce 1 kg of each product, together with the daily availability of each of the raw materials.

Product	Avex	Bumax	Ceedex	Dorax
Rostic	0.4	0.0	0.3	0.3
Senopure	0.4	0.2	0.3	0.1
Taltrate	0.0	0.4	0.2	0.4
Daily availability (kg)	200	120	250	150

Table $Q_3(a)$

The costs (in $ per kg) of each of the four raw materials are as given in table Q_3(b):

Raw material	Cost ($/kg)
Avex	15
Bumax	25
Ceedex	16
Dorax	20

Table Q_3(b)

The current selling prices of the three chemical products are given in table Q_3(c):

Product	Selling price ($/kg)
Rostic	35
Senopure	32
Taltrate	45

Table Q_3(c)

Rostic and Senopure are multi-purpose chemicals for which the market is in excess of the production capacity. Taltrate, however, is a special chemical and the maximum daily sales do not exceed 500kg.

3.1 Formulate a linear programming model to determine the daily production levels for each of the three chemicals to maximize the contribution to profits.
State briefly any assumptions you make.

(08 marks)

3.2 The final tableau of a Simplex solution to this problem is as given in table Q_3(d)

Basis	R	S	T	A	B	C	D	E	Value
S	0	1	0	1.875	2.5	0	-2.5	0	300
T	0	0	1	-0.9375	1.25	0	1.25	0	150
C	0	0	0	-0.5625	-0.25	1	-0.25	0	70
R	1	0	0	0.625	-2.5	0	2.5	0	200
E	0	0	0	0.9375	-1.25	0	-1.25	1	350
	0	0	0	15.6875	19.75	0	39.75	0	11,470

Table Q_3(d)

where R, S and T are the daily production levels of Rostic, Senopure and Taltrate respectively; A, B, C and D are the amounts of raw materials that fall short of the maximum available for Avex, Bumax, Ceedex, Dorax respectively and E is the amount of Taltrate short of its maximum demand.

Use the tableau to find the optimum daily production plan. State the resulting contribution to profit and the amount of any unused raw materials. *(04 marks)*

3.3 Write down the dual problem to the one you have formulated in part 3.1. State the solution to the dual problem and interpret your answers in the context of this question.

(04 marks)

3.4 A possibility exists for the company to manufacture a fourth product, Upelin, which would sell at \$50/kg. Each 1 kg of Upelin would require 0.5 kg of Bumax and 0.5 kg of Dorax.
Advise the chemical company as to whether they should manufacture Upelin.

(04 marks)

QUESTION 4

4.1 What is an assignment problem?

(03 marks)

4.2 Sky Airlines is a small air freight company based in Colombo and operating throughout the South Asia. The company has 6 aircraft of types as follows:

Aircraft Type	Availability
A	3
B	2
C	1

Operating costs and load carrying capacities of the aircrafts are as given in table Q_4(a).

Aircraft Type	Fixed cost (\$ per day)	Variable Cost (\$)		Capacity (tons)
		Per km	Per ton	
A	800	0.60	30	10
B	700	0.40	35	8
C	500	1.00	25	4

Table Q_4(a)

For any journey, variable costs are assigned on the basis of both distance flown and load being carried. All fixed and variable cost components are then added to obtain the relevant total cost.

On one particular day, there are 5 loads to be delivered to various destinations, the size of load and distances being given in table Q_4(b).

Load	Size (tons)	Distance (km)
1	10	200
2	2	550
3	8	320
4	5	280
5	3	450

Table Q_4(b)

The distances given above are direct from company's base in Colombo to the location involved and in each case there is no return load so that the aircraft will fly back empty. This means that mileage costs are incurred on both the outward and return journeys, but the tonnage cost is only incurred on the outward journey. All five return journeys can be completed within one day, and each aircraft can only fly one load in any one day. You may also assume that loads cannot be divided up and delivered in parts.

4.2.1 For each of the five loads, determine the cost of delivery using each of the feasible aircraft types.

(05 marks)

4.2.2 Setup the cost matrix for assigning each load to each particular aircraft. Using an appropriate procedure, decide which aircraft should be used for delivering each load so that total cost is minimized. What is the total cost of five deliveries?

(12 marks)

QUESTION 5.

A company is considering whether to lease a large factory, with a capacity of 100,000 units per annum, or a smaller factory, with a capacity of 70,000 units per annum for a manufacture of a new product, which is expected to have a life of 8 years. The company believes that there is a 40% chance that the demand will be 100,000 units per annum during the first two years, and a 60% chance that it will be 70,000 units per annum during this time. Demand for the subsequent six years is estimated as follow:

If annual demand for the first 2 years = 100,000 units

Annual demand for the next subsequent 6 years	Probability
100,000	0.3
70,000	0.5
50,000	0.2

If annual demand for the first 2 years = 70,000 units

Annual demand for the next subsequent 6 years	Probability
100,000	0.2
70,000	0.6
50,000	0.2

The fixed costs of operation are estimated as $ 50,000 per annum for the large factory and $40,000 per annum for smaller factory. Contribution per unit is expected to be $1 with the

large factory and $0.9 with the smaller factory. The company proposes to review the situation after two years, and would consider the following possible changes:

a) The enlargement of the smaller factory to a capacity of 100,000 units per annum if demand during the first two years is 100,000 units per annum This would add $12,000 per annum to fixed costs, but would not affect contribution per unit.
b) The reduction of the large factory to a capacity of 70,000 units per annum if demand during the first two years is 70,000 units per annum. This would save $8,000 per annum on fixed costs, but would not affect contribution per unit.

By considering and analysing a decision tree representing this situation, advice the company what course of action to pursue.

(20 marks)

QUESTION 6

6.1 What kind of policy or procedure would you recommend to improve the inventory operation in a department store?

(04 marks)

6.2 DB Plc operates a conventional stock control system based on re-order levels and Economic Order Quantities. The various control levels were set originally based on estimates which did not allow for any uncertainty and this has caused difficulties because, in practice, lead times, demands and other factors do vary.

As part of a review of the system, a typical stock unit, Unit No. X206 has been studies in details as follows.

Data for Unit No. X206

Lead times	**Probability**
15 working days	0.2
20 working days	0.5
25 working days	0.3

Demand per working day	**Probability**
5,000 units	0.5
7,000 units	0.5

It can be assumed that the demands would apply for the whole of the appropriate lead time.

DB Plc works for 240 days per year and it cost $0.15 per annum to carry a unit of X206 in stock. The re-order level for this part is currently 150,000 units and the re-order cost is $1,000.

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6.2.1 Calculate the level of buffer stock implicit in a re-order level of 150,000 units.

(04 marks)

6.2.2 Calculate the probability of a stock-out.

(02 marks)

6.2.3 Calculate the expected annual stock-outs in unit.

(04 marks)

6.2.4 Calculate the stock-out cost per unit at which it would be worthwhile raising the re-order level to 175,000 units.

(04 marks)

6.2.5 Discuss the possible alternatives to re-order level, EOQ inventory system and advantages and disadvantages.

(02 marks)

QUESTION 7

Ceylon Shipping Company has a warehouse terminal in Colombo dockyard. Each terminal dock has facility to accommodate maximum three trucks at a time. As trucks enter the terminal, the drivers receive numbers, and when one of the three dock space become available, the truck with the lowest number enters the vacant dock. Truck arrivals are Poisson distributed, and both unloading and loading times (service times) are exponentially distributed. The average arrival rate at the terminal is 5 trucks per hour, and the average service rate per dock is 2 trucks per hour.

7.1 Compute average number of trucks in the queue and average time a truck spends in the queue.

(04 marks)

7.2 The management of the shipping company is considering adding extra employees and equipment to improve the average service time per terminal dock to 25 minutes per truck. It would cost the company $18,000 per year to achieve this improved service. Management estimates that it will increase its profit by $750 per year for each minute it is able to reduce a truck's waiting time. Determine whether management should make the investment.

(04 marks)

7.3 Now suppose that the management of the shipping company has decided that truck waiting time is excessive and they want to reduce the waiting time. It has determined that there are two alternatives available for reducing the waiting time. They can add a fourth dock, or they can add extra employees and equipment at the exiting docks, which will reduce the average service time from the original 30 minutes to 23 minutes per truck. The costs of these alternatives are approximately equal. The management desires to implement the alternative that reduces waiting time by the greatest amount. Which alternative the management should select?

(Note: necessary formulae are provided at the end of this question paper)

(12 marks)

QUESTION 8

8.1 What is the essential feature of Monte Carlo Simulations?

(02 marks)

8.2 Briefly explain the role of computers in carrying out simulations.

(02 marks)

8.3 As a result of a routine analysis of cash flows, the Chief Accountant of Beta Chemicals International Limited considers that there are only three types of each cash flow which are likely to vary significantly from month to month.

They are: Wages and salaries
Raw materials purchases
Sales Revenue

Using data that have been collected over the last two years and taking into account likely changes in the level of operations during the next few months, the following distributions have been estimated for the monthly cash flow in each of these categories.

Wage and salaries ($'000)	Probability
10-12	0.3
12-14	0.5
14-16	0.2

Raw materials ($'000)	Probability
6-8	0.2
8-10	0.3
10-12	0.3
12-14	0.2

Sales revenue ($'000)	Probability
30-34	0.1
34-38	0.3
38-42	0.4
42-46	0.2

All other cash flows can be regarded as fixed, and amount to a net cash outflow of $14,000 per month. Currently Beta has cash assets of $50,000.

8.3.1 Using the random numbers given at the end of question 8, simulate six months' cash flows. You may assume that all cash flows are independent and take place at the end of month. From your simulation, estimate the probability of a net cash outflow in any month, and the cash balance at the end of 6 month period.
(Note Use mid-points as a representative figure for each class interval. Random Numbers are given at the end of question)

(10 marks)

8.3.2 What is the expected cash balance at the end of the 6 month period? Explain briefly why this value may be different from a corresponding value obtained from a simulation experiment.

(06 marks)

	Random numbers:					
Wages and Salaries	2	7	9	2	9	8
Raw materials	4	4	1	0	3	4
Sales revenue	0	6	6	8	0	2

END

NECESSARY FORMULAE FOR QUESTION NO. 7

	Single Server Model	Multiple Server Model
Average number of customers in the queuing system	$L = \frac{\lambda}{\mu - \lambda}$	$L = \frac{\lambda\mu(\lambda/\mu)^s}{(s-1)!(s\mu-\lambda)^2} P_o + \frac{\lambda}{\mu}$
Average number of customers in the queue	$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}$	$L_q = L - \frac{\lambda}{\mu}$
Average time a customer spends in the queuing system	$W = \frac{1}{\mu - \lambda}$	$W = \frac{L}{\lambda}$
Average time a customer spends in the queue	$W_q = \frac{\lambda}{\mu(\mu-\lambda)}$	$W_q = W - \frac{1}{\mu}$

λ = mean arrival rate;
μ = mean service rate;
s = number of servers

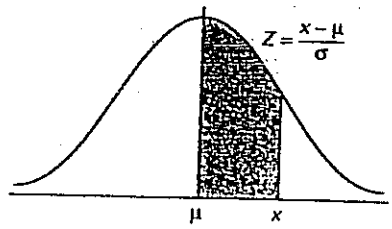

TABLE Normal Curve Areas

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

TABLE Selected Values of P_0 for the Multiple-Server Model

$\rho = \lambda/s\mu$	Number of Channels: s									
ρ	2	3	4	5	6	7	8	9	10	15
0.02	0.96079	0.94177	0.92312	0.90484	0.88692	0.86936	0.85215	0.83527	0.81873	0.74082
0.04	0.92308	0.88692	0.85215	0.81873	0.78663	0.75578	0.72615	0.69768	0.67032	0.54881
0.06	0.88679	0.83526	0.78663	0.74082	0.69768	0.65705	0.61878	0.58275	0.54881	0.40657
0.08	0.85185	0.78659	0.72615	0.67032	0.61878	0.57121	0.52729	0.48675	0.44983	0.30119
0.10	0.81818	0.74074	0.67031	0.60653	0.54881	0.49659	0.44933	0.40657	0.36788	0.22313
0.12	0.78571	0.69753	0.61876	0.54881	0.48675	0.43171	0.38289	0.33960	0.30119	0.16530
0.14	0.75439	0.65679	0.57116	0.49657	0.43171	0.37531	0.72628	0.28365	0.24660	0.12246
0.16	0.72414	0.61838	0.52720	0.44931	0.38289	0.32628	0.27804	0.23693	0.20190	0.09072
0.18	0.69492	0.58214	0.48660	0.40653	0.33959	0.28365	0.23693	0.19790	0.16530	0.06721
0.20	0.66667	0.54795	0.44910	0.36782	0.30118	0.24659	0.20189	0.16530	0.13534	0.04979
0.22	0.63934	0.51567	0.41445	0.33277	0.26711	0.21437	0.17204	0.13807	0.11080	0.03688
0.24	0.61290	0.48519	0.38244	0.30105	0.23688	0.18636	0.14660	0.11532	0.09072	0.02732
0.26	0.58730	0.45640	0.35284	0.27233	0.21007	0.16200	0.12492	0.09632	0.07427	0.02024
0.28	0.56250	0.42918	0.32548	0.24633	0.18628	0.14082	0.10645	0.08045	0.06081	0.01500
0.30	0.53846	0.40346	0.30017	0.22277	0.16517	0.12241	0.09070	0.06720	0.04978	0.01111
0.32	0.51515	0.37913	0.27676	0.20144	0.14644	0.10639	0.07728	0.05612	0.04076	0.00823
0.34	0.49254	0.35610	0.25510	0.18211	0.12981	0.09247	0.06584	0.04687	0.03337	0.00610
0.36	0.47059	0.33431	0.23505	0.16460	0.11505	0.08035	0.05609	0.03915	0.02732	0.00452
0.38	0.44928	0.31367	0.21649	0.14872	0.10195	0.06981	0.04778	0.03269	0.02236	0.00335
0.40	0.42857	0.29412	0.19929	0.13433	0.09032	0.06065	0.04069	0.02729	0.01830	0.00248
0.42	0.40845	0.27559	0.18336	0.12128	0.07998	0.05267	0.03465	0.02279	0.01498	0.00184
0.44	0.38889	0.25802	0.16860	0.10944	0.07080	0.04573	0.02950	0.01902	0.01225	0.00136
0.46	0.36986	0.24135	0.15491	0.09870	0.06265	0.03968	0.02511	0.01587	0.01003	0.00101
0.48	0.35135	0.22554	0.14221	0.08895	0.05540	0.03442	0.02136	0.01324	0.00826	0.00075
0.50	0.33333	0.21053	0.13043	0.08010	0.04896	0.02984	0.01816	0.01104	0.00671	0.00055
0.52	0.31579	0.19627	0.11951	0.07207	0.04323	0.02586	0.01544	0.00920	0.00548	0.00041
0.54	0.29870	0.18273	0.10936	0.06477	0.03814	0.02239	0.01311	0.00767	0.00448	0.00030
0.56	0.28205	0.16986	0.09994	0.05814	0.03362	0.01936	0.01113	0.00638	0.00366	0.00022
0.58	0.26582	0.15762	0.09119	0.05212	0.02959	0.01673	0.00943	0.00531	0.00298	0.00017
0.60	0.25000	0.14599	0.08306	0.04665	0.02601	0.01443	0.00799	0.00441	0.00243	0.00012
0.62	0.23457	0.13491	0.07550	0.04167	0.02282	0.01243	0.00675	0.00366	0.00198	0.00009
0.64	0.21951	0.12438	0.06847	0.03715	0.01999	0.01069	0.00570	0.00303	0.00161	0.00007
0.66	0.20482	0.11435	0.06194	0.03304	0.01746	0.00918	0.00480	0.00251	0.00131	0.00005
0.68	0.19048	0.10479	0.05587	0.02930	0.01522	0.00786	0.00404	0.00207	0.00106	0.00004
0.70	0.17647	0.09569	0.05021	0.02590	0.01322	0.00670	0.00338	0.00170	0.00085	0.00003
0.72	0.16279	0.08702	0.04495	0.02280	0.01144	0.00570	0.00283	0.00140	0.00069	0.00002
0.74	0.14943	0.07875	0.04006	0.01999	0.00986	0.00483	0.00235	0.00114	0.00055	0.00001
0.76	0.13636	0.07087	0.03550	0.01743	0.00845	0.00407	0.00195	0.00093	0.00044	0.00001
0.78	0.12360	0.06335	0.03125	0.01510	0.00721	0.00341	0.00160	0.00075	0.00035	0.00001
0.80	0.11111	0.05618	0.02730	0.01299	0.00610	0.00284	0.00131	0.00060	0.00028	0.00001
0.82	0.09890	0.04933	0.02362	0.01106	0.00511	0.00234	0.00106	0.00048	0.00022	0.00000
0.84	0.08696	0.04280	0.02019	0.00931	0.00423	0.00190	0.00085	0.00038	0.00017	0.00000
0.86	0.07527	0.03656	0.01700	0.00772	0.00345	0.00153	0.00067	0.00029	0.00013	0.00000
0.88	0.06383	0.03060	0.01403	0.00627	0.00276	0.00120	0.00052	0.00022	0.00010	0.00000
0.90	0.05263	0.02491	0.01126	0.00496	0.00215	0.00092	0.00039	0.00017	0.00007	0.00000
0.92	0.04167	0.01947	0.00867	0.00377	0.00162	0.00068	0.00028	0.00012	0.00005	0.00000
0.94	0.03093	0.01427	0.00627	0.00268	0.00113	0.00047	0.00019	0.00008	0.00003	0.00000
0.96	0.02041	0.00930	0.00403	0.00170	0.00073	0.00029	0.00012	0.00005	0.00002	0.00000
0.98	0.01010	0.00454	0.00191	0.00081	0.00033	0.00013	0.00005	0.00002	0.00001	0.00000