

**THE OPEN UNIVERSITY OF SRI LANKA**  
**FACULTY OF ENGINEERING TECHNOLOGY**  
**POSTGRADUATE DIPLOMA IN TECHNOLOGY IN INDUSTRIAL**  
**ENGINEERING – LEVEL 7**

**FINAL EXAMINATION – 2008/2009**

**MEX 7211 – OPERATION RESEARCH**

**DATE : 08 April 2009**

**TIME : 0930 hrs – 1230 hrs**

**DURATION: Three (03) hours**



**Answer any five (05) questions. All questions carry equal marks.**

- Q1. (a) A motorist observes that the distance he could cover with a tank full of petrol is normally distributed with mean 350km and standard deviation 50 km.
- (i) What is the probability that he could cover more than 400km with a tank full of petrol?
- (ii) What is the maximum distance he could cover with a tank full of petrol so that he can be 95% sure (certain) that he will not run out of petrol?
- (b) (i) Spark plugs are packed in boxes. Each box contains 10 plugs. The probability that a plug is defective is 0.1 and it is the same for all the plugs. What is the probability that 3 out of the 10 plugs are defective?
- (ii) A tutor observes that on the average 80% of the students pass the mathematics paper. He estimates the probability that 5 out of his 7 students will pass as

$${}^7C_5 (0.8)^5 (0.2)^2 = 0.275$$

Do you agree with this estimate? Give reasons.

- (c) A manager of a retail shop observes that on the average he receives two complaints a day. What is the probability that he will receive three complaints the next day?
- Q2. (a) (i) Explain the Bayes theorem
- (ii) Mr. Perera travels to office either by bus or train. Past records indicate that 60% of the time he has traveled by bus and 40% of the time he has traveled by train. The probability that he is late to office when he travels by bus or train are respectively 0.1 and 0.3
- (i) What is the probability that Mr. Perera is late to office?
- (ii) What is the probability that Mr. Perera traveled by train given that he is late to office?

- (b) A shop keeper claims that his car batteries last 24 months. To test this hypothesis a sample of 36 batteries were taken and the mean and standard deviation of life span is evaluated. It was observed that the mean was 19 hours with standard deviation 9 hours. Test the hypothesis that the shop keeper claim is correct at 5% level of significance.

- Q3. (a) A solar car company has a plant that manufacture family cars, station wagons and sports cars. The unit profit, variable production time and fixed cost for manufacturing these cars are explained in the table below.

Model	Unit Profit Rs. "000"	Variable production time (Hours)	Fixed Cost Rs. "000"
Family Cars	50	14	2000
Station Wagons	70	18	3000
Sports Cars	80	22	5000

Currently the car company has orders for 120 family cars 175 station wagon and 200 sports cars which must be satisfied. The car company wants to plan production so as to break even as quickly as possible. (That is cover the total fixed cost as quickly as possible) Formulate this as a linear programming problem.

- (b) Solve the following linear programming problem using simplex method  
Maximize  $P = 100x + 500y$

$$3x + 2y \leq 180$$

$$x + 2y \leq 80$$

$$y \leq 30$$

$$x \geq 0 \quad y \geq 0$$

- Q4. The road development department operate four worksites  $w_1, w_2, w_3$  and  $w_4$  that need to be supplied with bitumen. Their weekly requirement of bitumen is respectively 200, 500, 100 and 700 containers. This bitumen could be supplied by three suppliers  $s_1, s_2$  and  $s_3$  whose weekly capacities are respectively 600, 400 and 500 containers. The cost of transporting one container from a given supplier to a given worksite explained in the table below.

Suppliers	Worksites			
	Cost of transporting one container (Rs. '00)			
	$W_1$	$W_2$	$W_3$	$W_4$
$S_1$	12	7	15	8
$S_2$	5	11	9	14
$S_3$	10	16	12	17

The road development department wishes to find the transportation plan that would result in the least total transportation cost.

- a) Find an initial feasible solution.
- b) Solve the transportation problem.

Q5. Four research teams  $R_1, R_2, R_3$  and  $R_4$  are to be assigned to four research projects  $P_1, P_2, P_3$  and  $P_4$ . The time taken by a given research team to complete a given research project in months is explained in the table below.

	$P_1$	$P_2$	$P_3$	$P_4$
$R_1$	14	8	11	12
$R_2$	8	6	7	9
$R_3$	17	14	12	10
$R_4$	15	17	16	8

- a) Find the optimal assignment plan that would minimize the total time taken to complete all four projects.
- b) Find the optimal plan if a condition is laid down that research team  $R_4$  should not be assigned project  $P_4$ .

Q6. At a service station, vehicles arrive in a poisson fashion at the rate of 3 per hour. The service station has only one server and on the average it takes 15 minutes to service one vehicle. The service station works eight hours a day.

- (a)
  - (i) How many hours will the server idle per day? or What percentage of time is server idle?
  - (ii) How many vehicles would you expect to see on the average at the service station?
  - (iii) What would be the average time that a vehicle must wait to get service?
  - (iv) What is the probability that when a vehicle arrives, it will be taken for servicing without having to wait.
- (b) What is the probability that when a vehicle arrives it will be taken for servicing without having to wait if there are two servers instead of one?

Q7. A motor car industry observes that the annual demand for one of the components used is 9000. The cost of placing a order for this component is Rs. 2400/-. The cost of holding one item in stock for one year is Rs. 30/-.

- (a) If stock outs are not allowed, calculate;
  - (i) Economic order quantity
  - (ii) Total inventory cost
  - (iii) Re-order-level if lead time is one month
  - (iv) Re-order-level if lead time is four months

- (b) If stock outs are allowed and stock out cost is Rs. 50/- per item per year calculate;
- Economic order quantity
  - Maximum level of stock
- (c) The industry decides to buy a machine that could turn out this component. The production capacity of the machine is 21,000 per annum. The set-up cost is 800.00. Calculate;
- Economic order quantity.

Q8. An educational institute wishes to conduct either computer classes, spoken English classes or revision classes next year. The institute is planning to maximize their profit however the profit would depend on the performance at the forthcoming G C E (AL) examination. For example a poor performance would give more scope for revision classes rather than computer classes. The performance at the G.C.E (AL) Examination could be identified as "Good", "Average" or "Bad". The expected profit for the three decisions under different levels of performance is explained in the table below.

Expected profit Rs. "000"

Decision Alternatives	Performance at G C E (AL) Examination		
	Good	Average	Bad
Computer classes	85	75	25
Spoken English classes	70	65	45
Revision classes	15	50	70

- (a)
- What is the best decision if it is certain that the performance will be good?
  - What is the best decision under pessimistic approach (Max Min)?
  - What is the best decision under optimistic approach (Max Max)?
  - What is the best decision based on minimizing total regret?
- (b) It is estimated that the probability that performance at G.C.E (AL) is good, average or bad is respectively 0.4, 0.5 and 0.1.
- Evaluate best decision based on (EMV) (Expected monetary value)
  - Evaluate best decision based on (EOL) (Expected opportunity loss)
  - Evaluate E.V.P.I (Expected value of perfect information)

NECESSARY FORMULAE FOR QUESTIONS 6 AND 7.

1) Single server queue

(i) Server idle time =  $H(1 - \theta)$

(ii)  $L_s = \frac{\theta}{(1 - \theta)}$

(iii)  $L_s = \lambda W_s$

(iv)  $P_{(n)} = \theta^n (1 - \theta)$

2) Multi-server queue

(i)  $P(n) = \frac{\theta^n}{n!} P(0)$  for  $n < s$

(ii)  $P(n) = \frac{S^s}{S!} \left(\frac{\theta}{S}\right)^n P(0)$  for  $n \geq S$

(iii)  $\frac{1}{P(0)} = \left[ \sum_{n=0}^{s-1} \frac{\theta^n}{n!} + \frac{S^s}{S!} \left(\frac{\theta}{S}\right)^s \left(\frac{1}{1 - \theta/S}\right) \right]$

3) Inventory control

(i)  $EOQ = \sqrt{\frac{2DA}{C}}$

(ii)  $K = \frac{DA}{Q} + \frac{1}{2}QC$

(iii)  $EOQ = \sqrt{\frac{2DA}{C} \left(\frac{C+S}{S}\right)}$

(iv)  $Q = \frac{S \times EOQ}{(C+S)}$

(v)  $EOQ = \sqrt{\frac{2DA}{C \left(1 - \frac{D}{R}\right)}}$