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THE OPEN UNIVERSITY OF SRI LANKA
FACULTY OF ENGINEERING TECHNOLOGY
POSTGRADUATE DIPLOMA IN TECHNOLOGY IN INDUSTRIAL ENGINEERING – LEVEL 7
FINAL EXAMINATION – 2009/2010

MEX 7211– OPERATIONS RESEARCH

DATE : *01 April 2010*

TIME : *1400 hrs – 1700 hrs*

DURATION: *Three (03) hours*

Answer any five (05) questions. All questions carry equal marks.
Normal distribution tables and graph papers are provided.

Q1. (a) It is observed that the speed driven by motorists on interstate highways is normally distributed with mean 60km/h and standard deviation 20 km/h.

(i) What percentage or motorists will drive between the speeds 40km/h and 70 km/h.?

(ii) If the maximum speed limit is 90 km/h what percentage of motorists will violate the speed limit?

(b) The life time of a certain manufacture's washing machine is normally distributed with mean 4 years. Only 15% of these machines last at least 5 years. What is the standard deviation of the life time of the washing machines?

(c) Past experience indicates that 30% of all individuals entering a certain store decide to make a purchase. What is the probability that at least two out of group of five individuals who just entered would make a purchase?

Q2. (a) In a certain state, 15% of the vehicles have high emission of carbon dioxide, 25% have moderate emission while 60% have low emission of carbon dioxide. It is observed that 10% of the vehicles with high emission pass the vehicle emission test while 60% vehicles with moderate emission and 90% of vehicles with low emission pass the vehicle emission test.

(i) What is the probability that a vehicle that just drives in would pass the vehicle emission test?

(ii) Given that a vehicle has passed the vehicle emission test what is the probability that the vehicle has low emission of carbon dioxide?

(b) The following table gives information on age and the fuel consumed to run 100km. of eight vehicles.

Age (Years)	Fuel consumed to run 100km (liters)
4	7
10	8
20	14
15	12
7	8
6	6
8	8
12	11

(i) Calculate the correlation coefficient between age and fuel consumed.
(ii) Develop the regression equation of the form $y = a + bx$ where "x" is age and "y" is fuel consumed.

(iii) Predict the fuel consumption of a vehicle with age 18 years.

Q3. (a) An organisation produces two products "A" and "B" that gives a unit profit of Rs. 35 and Rs. 29 respectively. They use raw materials and machine hours as resources. The organization has 2000kg of raw material and 1800 machine hours for their weekly production. The quantity of raw materials and machine hours required to produce one unit of each type of product is explained in the table below. Find how many items of each product should be produced to maximize profit.

Product	Unit Requirement	
	Raw Materials (Kg)	Machine Hours
A	20	30
B	40	20

(i) Formulate the linear programming model.
(ii) Solve the problem using graphical method
(iii) If the availability of raw materials was increased to 2400 kg what would the new optimal solution be?

(b) The following table is the final incomplete simplex table of a maximization problem.

c_j / c_i	Basis	Solution	4 X_1	6 X_2	S_1	S_2
	X_2	1			¼	-1/8
	X_1	3			-1/4	-3/8
	Z_j					
	$C_j - Z_j$					

(i) Copy down the table and complete it by filling the blank cages.
(ii) Write down the objective function.
(iii) Is the solution feasible (give reasons).
(iv) Is the solution optimal (give reasons).
(v) Write down the optimal solution.
(vi) Has the problem got multiple optimal solutions (Give reasons).

Q4. Four men M_1, M_2, M_3 and M_4 can complete any of the four jobs J_1 , J_2, J_3 and J_4 but the time they take to complete the jobs differ among the four men as shown in the table below.

Time taken to complete job (Days)

Men	Job			
	J_1	J_2	J_3	J_4
M_1	17	14	19	12
M_2	15	10	18	11
M_3	7	9	8	6
M_4	22	17	21	19

(i) Use assignment theory to find how the men should be assigned to the jobs so that the total time taken to complete all four jobs is a minimum.

(ii) If the four men work on the four jobs simultaneously then how should the men be assigned to the jobs so that all four jobs are completed in the earliest possible time (Bottle – Neck assignment problem)

Q5. Goods are to be transported from three stores "A", "B" and "C" to four sales outlets P, Q, R and S. The weekly capacities of the stores A, B and C are respectively 22, 15 and 8 units. The weekly demand of the sales outlets P, Q, R and S are respectively 7, 12,17and 9 units. Cost of transporting one unit from a given store to a given sales out let is explained in the table below.

Cost of Transport (Rs)

Store	Sales Out Let			
	P	Q	R	S
A	6	3	5	4
B	5	9	2	7
C	5	7	8	6

It is intended to find the transportation plan that would minimize total transportation cost.

(i) Use least cost method to find an initial feasible solution

(ii) Solve the transportation problem using MODI-method.

Q6. A service station has only one servicing plant and six parking lots to park vehicles. When parking space is full vehicles are parked on the high-way. Vehicles arrive in a poisson fashion at the rate of 7 per day. The time taken to service has a negative exponential distribution with mean 75 minutes. The service station works 10 hours a day.

(i) What is the probability that there are no vehicles at the service station?
(ii) How many hours in the day would the service station idle?
(iii) What is the probability that a vehicle is parked on the high-way?
(iv) What is the probability that a vehicle is parked on the high-way if another servicing plant is installed. It is assumed that this additional servicing plant will use up the space of three parking lots.

(v) Traffic police is unhappy about vehicles being parked on the high-way and request that the probability that a vehicle is parked on the high way be kept below 1% (or 0.01)

The service station has two alternatives "a" and "b" to meet this request

a) Purchase land and have more parking lots.

b) Have two servicing plants instead of one.

There is enough land available to purchase. However if this alternative is decided, each additional parking lot would cost Rs. 30,000. The cost of installing a second servicing plant is Rs. 500,000. Which alternative would you select from an economic point of view?

Q7. An Educational institute hopes to conduct either computer classes, spoken English classes, or revision classes during the next three months. The institute wishes to maximize their profit. However the profit would depend on the performance at the forthcoming GCE (A/L) Examination. For example a poor performance would give more scope for revision classes rather than computer classes. The performance is identified as "Good" "Average" or "Bad" and they explain the states of nature. The expected profit that is earned for the three decisions under different levels of performance is explained in the table below.

Profit Earned Rs. "000"

Decision Alternative	Performance at GCE (AL)		
	Good	Average	BAD
Computer Classes	85	75	25
Spoken English Classes	70	65	45
Revision Classes	15	50	70

(i) What is the best decision if it is certain that the performance will be bad?
(ii) What is the best decision under the pessimistic approach?
(iii) What is the best decision under the optimistic approach?
(iv) What is the best decision based on minimizing total regret?

(v) If it is known that the probability of good, average and bad performance is respectively 0.5, 0.3 and 0.2.
   a) Construct a decision tree.
   b) Find the best decision based on expected monotory vale (EMV)
   c) Find the best decision based on expected opportunity loss (EOL)

(vi) Calculate the expected valve of perfect information (EVPI).

Multi Server Queue

$$P(n) = \frac{\theta^n}{n!} P(O) \text{ when } (n<S)$$

$$P(n) = \frac{S^s}{S!}\left(\frac{\theta}{S}\right)^n P(O) \text{ when } (n \geq S)$$

$$P(O)\left[\sum_{n=0}^{(s-1)}\frac{\theta^n}{n!} + \frac{S^s}{S!}\left(\frac{\theta}{S}\right)^s\left(\frac{1}{1-\theta/S}\right)\right] = 1$$

Linear Regression

In the equation $y = a + bx$

$$b = \frac{n\sum xy - \left(\sum x\right)\sum y}{n\sum x^2 - \left(\sum x\right)^2}$$

$$a = \frac{\sum y}{n} - b\frac{\sum x}{n}$$

Summary

Variables :

λ = Rate of arrival of units
μ = Rate of service completion
$\theta = \lambda/\mu$
H = Number of working hours per day.
$P_{(n)}$ = Probability of "n" units in the queuing system
L_s = Average number of units in queuing system
L_q = Average number of units in queue
W_s = Average time spent by unit in queuing system
W_q = Average time spent by unit in queue.

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Formulae

$P(n) = \theta P(n-1)$ ---------------(1)

$P(n) = \theta^n P(0)$ ---------------(2)

$P(n) = \theta^n (1-\theta)$ ---------------(3)

$$\begin{bmatrix} \text{Probability that} \\ \text{queuing system empty} \end{bmatrix} = (1-\theta) \text{ ------------------(4)}$$

$$\begin{bmatrix} \text{Probability that} \\ \text{the server is idle} \end{bmatrix} = (1-\theta) \text{ ----------------(5)}$$

$$\begin{bmatrix} \text{Number of hours} \\ \text{server idle per day} \end{bmatrix} = H(1-\theta) \text{ -------------------(6)}$$

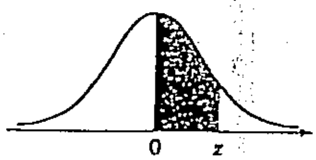
$L_s = \theta / (1-\theta)$ ----------------(7)

$L_q = \theta^2 / (1-\theta)$ ----------------(8)

$L_s = \lambda\ Ws$ --------------------(9)

$L_q = \lambda\ Wq$ --------------------(10)

Normal Curve Areas

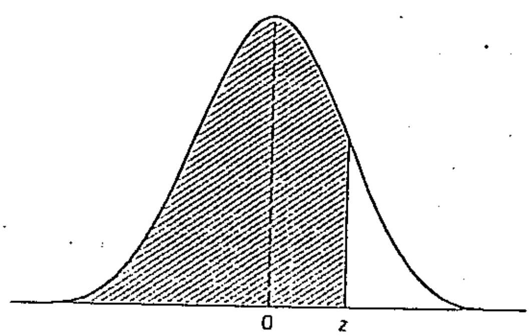


z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Source: Abridged from Table II of A. Hald, *Statistical Tables and Formulas* (New York: John Wiley & Sons, Inc.), 1952. Reproduced by permission of A. Hald and the publisher.

Table A2. Values of z, the standard normal variable, from 0·0 by steps of 0·01 to 3·9, showing the cumulative probability up to z.
(Probability correct to 4 decimal places).

z	0·00	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09
0·0	·5000	·5040	·5080	·5120	·5160	·5199	·5239	·5279	·5319	·5359
·1	·5398	·5438	·5478	·5517	·5557	·5596	·5636	·5675	·5714	·5753
·2	·5793	·5832	·5871	·5910	·5948	·5987	·6026	·6064	·6103	·6141
·3	·6179	·6217	·6255	·6293	·6331	·6368	·6406	·6443	·6480	·6517
·4	·6554	·6591	·6628	·6664	·6700	·6736	·6772	·6808	·6844	·6879
·5	·6915	·6950	·6985	·7019	·7054	·7088	·7123	·7157	·7190	·7224
·6	·7257	·7291	·7324	·7357	·7389	·7422	·7454	·7486	·7517	·7549
·7	·7580	·7611	·7642	·7673	·7704	·7734	·7764	·7794	·7823	·7852
·8	·7881	·7910	·7939	·7967	·7995	·8023	·8051	·8078	·8106	·8133
·9	·8159	·8186	·8212	·8238	·8264	·8289	·8315	·8340	·8365	·8389
1·0	·8413	·8438	·8461	·8485	·8508	·8531	·8554	·8577	·8599	·8621
·1	·8643	·8665	·8686	·8708	·8729	·8749	·8770	·8790	·8810	·8830
·2	·8849	·8869	·8888	·8907	·8925	·8944	·8962	·8980	·8997	·9015
·3	·9032	·9049	·9066	·9082	·9099	·9115	·9131	·9147	·9162	·9177
·4	·9192	·9207	·9222	·9236	·9251	·9265	·9279	·9292	·9306	·9319
·5	·9332	·9345	·9357	·9370	·9382	·9394	·9406	·9418	·9429	·9441
·6	·9452	·9463	·9474	·9484	·9495	·9505	·9515	·9525	·9535	·9545
·7	·9554	·9564	·9573	·9582	·9591	·9599	·9608	·9616	·9625	·9633
·8	·9641	·9649	·9656	·9664	·9671	·9678	·9686	·9693	·9699	·9706
·9	·9713	·9719	·9726	·9732	·9738	·9744	·9750	·9756	·9761	·9767
2·0	·9772	·9778	·9783	·9788	·9793	·9798	·9803	·9808	·9812	·9817
·1	·9821	·9826	·9830	·9834	·9838	·9842	·9846	·9850	·9854	·9857
·2	·9861	·9864	·9868	·9871	·9875	·9878	·9881	·9884	·9887	·9890
·3	·9893	·9896	·9898	·9901	·9904	·9906	·9909	·9911	·9913	·9916
·4	·9918	·9920	·9922	·9925	·9927	·9929	·9931	·9932	·9934	·9936
·5	·9938	·9940	·9941	·9943	·9945	·9946	·9948	·9949	·9951	·9952
·6	·9953	·9955	·9956	·9957	·9959	·9960	·9961	·9962	·9963	·9964
·7	·9965	·9966	·9967	·9968	·9969	·9970	·9971	·9972	·9973	·9974
·8	·9974	·9975	·9976	·9977	·9977	·9978	·9979	·9979	·9980	·9981
·9	·9981	·9982	·9982	·9983	·9984	·9984	·9985	·9985	·9986	·9986
3·0	·9987	·9987	·9987	·9988	·9988	·9989	·9989	·9989	·9990	·9990
·1	·9990	·9991	·9991	·9991	·9992	·9992	·9992	·9992	·9993	·9993
·2	·9993	·9993	·9994	·9994	·9994	·9994	·9994	·9995	·9995	·9995
·3	·9995	·9995	·9995	·9996	·9996	·9996	·9996	·9996	·9996	·9997
·4	·9997	·9997	·9997	·9997	·9997	·9997	·9997	·9997	·9997	·9998
·5	·9998	·9998	·9998	·9998	·9998	·9998	·9998	·9998	·9998	·9998
·6	·9998	·9998	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999
·7	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999
·8	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999
·9	1·0000									



The curve is $N(0, 1)$, the standard normal variable. The table entry is the shaded area $\Phi(z) = \Pr(Z < z)$. For example, when $z = 1{\cdot}96$ the shaded area is 0·9750. Critical values of the standard normal distribution will be found in the bottom row of Table A3.