



**THE OPEN UNIVERSITY OF SRI LANKA**  
**FACULTY OF ENGINEERING TECHNOLOGY**  
**POST GRADUATE DEIPLOMA IN TECHNOLOGY IN INDUSTRIAL ENGINEERING – LEVEL 7**  
**FINAL EXAMINATION – 2010/2011**  
**MEX 7211 – OPERATION RESEARCH**  
**DATE : 31<sup>ST</sup> MARCH 2001**  
**TIME : 1400 - 1700 HRS**  
**DURATION : 3 HOURS**  
**ANSWER ANY FIVE (05) QUESTIONS**

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- Q1. a) A six sided dice is thrown twice. What is the probability that the sum of the results is five?
- b) Three players A, B and C toss a coin in turn starting with A. The first to throw a head is the winner. What are their respective chances of winning?
- c) (i) State Bayes theorem.  
(ii) Suppose that in answering a question in a multiple choice test an examinee either knows the answer with probability "P" or he guesses with probability "(1-P)" Assume that the probability of answering a question correctly is ONE for an examinee who knows the answer and  $1/m$  for the examinee who guesses where "m" is the number of multiple choice alternatives. Show that the probability that the examinee knew the answer given that he has correctly answered is
- $$\frac{mp}{1 + (m - 1)p}$$
- (iii) In the above example if it is assumed that it is equally likely that the examinee knew or guessed the answer, find the least number of multiple choice alternatives required to ensure that the probability that examinee knew the answer given that he has answered correctly is at least 0.8.
- Q2. a) Spark plugs are packed in boxes, eight in each box. The probability that a spark plug is defective is 0.15.  
(i) What is the probability that exactly two plugs in a box are defective?  
(ii) What is the probability that none of the plugs in the box are defective?  
(iii) If five such boxes are randomly selected what is the probability that exactly two boxes will have none of the plugs defective?
- b) At a police station it is observed that on the average they receive two complaints every hour.  
(i) What is the probability that they will receive 3 complaints the next hour?  
(ii) What is the probability that they will receive 3 complaints within the next two hours?

- c) The time taken by an employee to travel from home to office is normally distributed with mean 60 minutes and standard deviation 15 minutes if his work place starts work at 9.00 am. At what time should he leave home to be 95% sure that he will not be late to office?

- Q3. The following table describes the daily output and the number of years of service of eight operators.

Operator Number	Number of years of service	Daily output "Rs. 000"
1	7	1.0
2	12	2.0
3	25	4.0
4	5	0.6
5	10	1.8
6	30	4.2
7	20	3.6
8	15	3.0

- (i) Calculate the correlation coefficient between years of service and daily output.
  - (ii) Develop the regression equation of the form  $y = a + bx$  where "y" is output and "x" is years of service.
  - (iii) Estimate the daily output of an operator with 18 years of service.
  - (iv) Calculate the sum of squares error given as "SSE"
  - (v) Find the coefficient of determination.
  - (vi) Calculate the standard error of the "b" coefficient given by  $S_b$ .
- Q4. At a laboratory blood samples are received in a poisson fashion at the rate of 8 per hour. There is only one technologist who on the average takes 6 minutes to test one sample. The laboratory works 10 hours per day
- (i) How many hours does the technologist idle per day?
  - (ii) What is the probability that there are three samples at the laboratory?
  - (iii) On the average how many blood samples are there at the laboratory?
  - (iv) On the average how long must a sample be kept at the laboratory?
  - (v) On the average how many sample are there waiting to be tested?
  - (vi) On the average how long must a blood sample be kept until it is taken for the test?
  - (vii) Blood samples, when they are received are kept in a freezer that has the capacity to store 8 samples. When a sample is received, if the freezer is full the sample is kept out side. What is the probability that a sample just received is kept out side the freezer?
  - (viii) What is the probability that a sample just received is kept outside the freezer if another freezer of the sample capacity is purchased?

- Q5. A garment factory has just completed their new building with a floor space of 1080 sq ft and they need to purchase machines for this new complex. There are two types of machines available namely type (A) and type (B). A type (A) machine would cost Rs. 8000, use 18 sq ft of floor space and give a daily output of 5000. It consumes 9 power units to produce 5,000 garment items.. A type (B) machine would cost Rs. 10,000, use 6 sq ft of floor space and give a daily out put of 3000. It consumes 7 power units to produce 3,000 garment items.. There is Rs. 800,000 of capital available to purchase machines. The tariff employed by the electricity board is such that when the daily consumption exceed 630 power units, the bill is doubled. It is therefore necessary to keep the daily power consumption below 630 units. These details are further explained in the Table below. The Management of the garment factory hopes to maximize total output.

Type of machine	Resource Requirement per machine			Daily output ("000" Items)
	Capital (Rs 000)	Floor Space (sq ft)	Power (Units)	
Type (A)	8	18	9	5
Type (B)	10	6	7	3
Total Availability of Resources	800	1080	630	

- (i) Formulate the linear programming model.
  - (ii) Solve the problem using graphical method.
  - (iii) With the help of the graph find the range of values of the " $C_j$ " constants (Namely 5 and 3) for which the optimal solution will remain optimal.
- Q6. A trader engaged in selling car batteries observes that the annual demand for his batteries is 720. The cost of a battery is Rs. 3000. The cost of placing one order for batteries is Rs. 1500 and the cost of holding one battery in stock for one year is Rs. 96. It is assumed that stock outs are not allowed.
- a) Calculate the Economic Order Quantity (EOQ).
  - b) Calculate the minimum inventory cost corresponding to EOQ.
  - c) Calculate Re-order level (ROL) if lead time is one month.
  - d) Calculate Re-order level (ROL) if lead month is three months.
  - e) Find the economic order quantity if the shelf space occupied by a battery is 30 sq inches and the total shelf space available is 6000 sq inches.
  - f) Find the economic order quantity if the capital available to purchase batteries is Rs. 400,000.
  - g) Find the economic order quantity if a 5% discount is given to all purchases more than 100 batteries.

h) Suppose it is assumed that stock outs are allowed and the cost of running one battery out of stock for one year is Rs. 32 then

- (i) Calculate economic order quantity (EOQ)
- (ii) Calculate the maximum level of stock.

Q7. The road development department operate four work sites  $W_1, W_2, W_3$  and  $W_4$  that need to be supplied with bitumen. Their weekly requirement of bitumen is respectively 200, 500, 100 and 700 containers. The bitumen could be supplied by three suppliers  $S_1, S_2$  and  $S_3$  whose weekly capacities are respectively 600, 400 and 500 containers. The cost of transporting one container from a given supplier to a given worksite is explained in the table below.

Cost of transporting one container (Rs 000)

	$W_1$	$W_2$	$W_3$	$W_4$
$S_1$	12	7	15	8
$S_2$	5	11	9	14
$S_3$	10	16	12	17

The road development department wishes to find the transportation plan that would result in the least total transportation cost

- (i) Find an initial feasible solution.
- (ii) Solve the transportation problem.
- (iii) Suppose it is necessary that we purchase the containers and then transport and the purchasing unit prices of  $S_1, S_2$  and  $S_3$  are respectively 2, 1 and 3, show that your optimal solution is not changed.

Q8. Five men  $M_1, M_2, M_3, M_4$  and  $M_5$  can perform any of the five jobs  $J_1, J_2, J_3, J_4$  and  $J_5$  but the time they take to complete jobs differ as explained in the table.

TIME TAKEN TO COMPLETE JOB (DAYS)

Men	Jobs				
	$J_1$	$J_2$	$J_3$	$J_4$	$J_5$
$M_1$	13	5	9	18	12
$M_2$	13	19	6	13	14
$M_3$	3	2	4	4	5
$M_4$	18	9	13	20	16
$M_5$	12	6	14	19	10

Find how the five men be assigned to the five jobs so that the total time taken to complete all five jobs is a minimum.

Mathematical FormulaeBinomial -  ${}^n C_r p^r q^{n-r}$ Poisson -  $e^{-a} \frac{a^x}{x!}$ Queuing Theory

$$P(x) = \theta^n (1 - \theta)$$

Server Idle Time =  $H(1-\theta)$ 

$$L_s = \frac{\theta}{1-\theta} \quad L_q = \frac{\theta^2}{1-\theta}$$

$$L_s = \lambda W_s \quad L_q = \lambda W_q$$

Correlation and Regression

$$r = \frac{\Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}}{\sqrt{\left[\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right] \left[\Sigma y^2 - \frac{(\Sigma y)^2}{n}\right]}}$$

$$b = \frac{n \Sigma xy - (\Sigma x)(\Sigma y)}{n \Sigma x^2 - (\Sigma x)^2} \quad a = \frac{\Sigma y}{n} - b \frac{\Sigma x}{n}$$

$$SEE = \Sigma y_i^2 - a \Sigma y_i - b \Sigma x_i y_i$$

$$S_b = \frac{\sqrt{SEE}}{(n-2)} / \sqrt{SSX} \quad SSX = \Sigma X_R^2 - NX^{-2}$$

Inventory control

$$EOQ = \sqrt{\frac{2DA}{C}} \quad k = \frac{DA}{Q} + \frac{1}{2} QC$$

$$EOQ = \sqrt{\frac{2DA}{C} \left(\frac{C+S}{S}\right)} \quad \text{--- (With Stock Outs)}$$

$$a = \frac{S \times EOQ}{(C+S)}$$

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