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The Open University of Sri Lanka

Faculty of Engineering Technology

Study Programme	: Master of Technology in Industrial Engineering - LEVEL 07
Name of the Examination	: Final Examination
Course Code and title	: MEX 7211 - OPERATIONS RESEARCH
Academic Year	: 2015/2016
Date	: 24th November , ২০১৬
Time	: 09.30 – 12.30 hrs
Duration	: 3 hours

General Instructions

1. Read all instruction carefully before answering the questions.
2. This question paper consists of 8 questions. All questions carry equal marks.
3. Answer any 5 questions only.
4. Graph paper will be provided.
5. Normal distribution tables will be provided.
6. Mathematical formula will be given.

1)

a) 600 students were interviewed to assess their computer skills. A breakdown of computer skills by gender is shown in the table below.

Gender	Computer Skills			Total
	Low	Average	High	
Male	50	70	80	200
Female	100	125	175	400
Total	150	195	255	600

A student is selected at random. Find the following probabilities.

(i) P (male) (ii) $P\left(\frac{\text{Low}}{\text{skills}}\right)$ (iii) $P\left(\text{male} \cap \frac{\text{Low}}{\text{Skills}}\right)$

(iv) $P\left(\frac{\text{Feamle}}{\text{High Skills}}\right)$ (v) $P\left(\frac{\text{High Skills}}{\text{Female}}\right)$

b)

The financial manager of an organization is concerned about payment of bills by his clients, he has instructed his officers to give telephone calls to clients at least three weeks prior to the due date. However due to pressure of work the officers can contact by phone only 80%of the clients. On the average 60% of the client who have received telephone calls makes payment while 40% of clients who do not receive telephone calls make payment on time. A client is selected at random.

(i) What is the probability that the client has made the payment?

(ii) Given that the client has made the payment what is the probability that he received a telephone call.

(iii) Given that the client has not made the payment what is the probability that he has received a telephone call.

2)

It is suggested that the number of break downs of a machine is related to its age. Data on age donated by "x" and number of break downs per year denoted by "y" from six machine is described below. The values of x^2, y^2 and xy are also given below.

Machine No.	**Age (x)**	**Break downs (y)**	x^2	y^2	xy
1	9	14	81	196	126
2	2	3	4	9	6
3	5	4	25	16	20
4	7	9	49	81	63
5	6	11	36	121	66
6	8	8	64	64	64
Total	**37**	**49**	**259**	**487**	**345**

(i) Calculate the correlation coefficient between age and machine break downs.

(ii) Evaluate the line of regression of the form $y = a + bx$

(iii) Estimate the number of break downs per year of a machine with age 10 years.

(iv) What is the residual of the observation, where $x = 5$

(v) Evaluate the sum of squares error given as "SSE"

(vi) Calculate the coefficients of determination (R^2) and interpret the results.

(vii) Calculate the standard error of the "b" coefficient given by SBI.

(viii) Develop a 95% confidence interval estimate for the "b" coefficient.

3)

Anura land sale have purchased a large acreage of land and is planning to block out and sell. They plan to sell either 15 Perch blocks, 25 Perch blocks or 40 Perch blocks. How ever the profit that could be made depends on the demand for residential land which could be identified as Low, Average and High. The profit that could be derived for a given decision under each level of demand is shown in table below.

Profit Rs. "000"

Decision alternative	Demand for residential land		
	Low	Avg.	High
15 Perch	80	60	90
25 Perch	70	80	120
40 Perch	20	90	150

(i) Evaluate the best decision based on

a) Pessimistic Approach

b) Optimistic Approach

c) Minimizing Total Regret

(ii) If it is known that the probability of demand for land being Low, Avg. and High are respectively 0.4, 0.3 and 0.3. Find the best decision based on Expected Monetary Value (EMV).

(iii) Find best decision based on Expected Opportunity Loss (EOL)

4)

a)

State the condition for equilibrium in a single server queue.

b)

At the petrol station there is only one unit to pump petrol and vehicles arrive in a Poisson fashion at the rate of 10 per hour. The average time taken to pump Petrol to one vehicle is 5 minutes and has a Negative Exponential Distribution. The petrol station works 24 hours a day.

(i) What is the probability that there are three vehicles at the Petrol station?

(ii) How many hours does the server at the Petrol station idle per day?

(iii) On average how many vehicles are there at the Petrol station?

(iv) On average how many vehicle are there waiting to pump Petrol?

(v) On average how long must a vehicle wait at the station to pump Petrol?

(vi) On average how long must a vehicle wait until it is taken to pump Petrol?

5)

a)

Bakelands Ltd. produces two bakery products namely product (A) and Product (B). While all other resources are available in plenty both the products use flour and oven space as scarce resources. For a given period there is only 240 Kg of flour and 315 sqft of oven space available. One case of product (A) requires 8 Kg of flour, 5 sqft of oven space and gives a profit of Rs. 3000. One case of product (B) requires 3 Kg of flour 6 sqft of oven space and gives a profit of Rs. 2000. A summary of these details are described below. Bakelands hopes to maximize profit and wants to know how many case of product (A) and product (B) that should be produced.

Product	**Resource requirement per case**		**Profit Rs."000"**
	Flour (Kg)	**Oven space (sqft)**	
Product (A)	8	5	3
Product (B)	3	6	2
Total available of resource	240	315	

(i) Solve the liner programming problem using simplex method.

(ii) Write down the dual of this problem.

(iii) State the solution of the dual problem.

(iv) Due to a sudden change in the market the availability of flour is expected to drop. By how much should it drop to change the optimal solution.

b)

The following table is the final incomplete simplex table of a maximizing problem.

C_j / C_b	**Basis**	**Solution**	**120**	**150**		
			X_1	X_2	S_1	S_2
	X_2	60			2/3	-1/2
	X_1	30			-1/3	1/2
	Z_j					
	C_j-Z_j					

(i) Copy down the table and complete it filling the blank cells.
(ii) Write down the objective function of the problem.
(iii) Is the solution feasible? Give reasons.
(iv) Is the solution optimal? Give reasons.
(v) Has this problem got multiple optimal solutions? Give reasons.
(vi) Write down the optimal solution.
(vii) Write down the shadow prices of the resources S_1 and S_2 and explain its meaning.

6)

A retailer observes that the annul demand for his tyres is 3000. The cost of placing one order for tyres is Rs. 1350/=. The inventory holding cost of one tyre for a period of one year is Rs. 90/=. It is assumed that stock outs are not allowed.

a)

(i) Calculate EOQ (Economic Order Quantity)
(ii) Calculate Re-order level (ROL) if lead time is one month.
(iii) Calculate Re-order level (ROL) if lead time is two months.
(iv) State your assumptions.

b)

Due to change in market conditions the stock out cost has come down. The cost of having one tyre out of stock for one year is Rs. 160/=. If stock out are allowed;

(i) Calculate EOQ (Economic Order Quantity)
(ii) Calculate the maximum level of stock.

7)

a)

State the limitations of assignment theory.

b)

Tender have being called for four projects, P_1, P_2, P_3 and P_4 from four contractors, C_1, C_2, C_3 and C_4 who are genuine and has kept a good past record. Each contractor has quoted for all four projects and their quotations differ as shown in the table below.

Quotation in Rs. "000"

Contractor	Projects			
	P_1	P_2	P_3	P_4
C_1	17	14	12	18
C_2	11	7	9	8
C_3	21	27	15	12
C_4	20	17	14	16

(i) Using the assignment theory find how the projects should be assigned among the contractors such that the total cost of all four projects is a minimum.

(ii) Find how the projects be assigned if there is a condition to say that project "P_2" should not be given to contractor "C_2".

8)

Write short notes on the following.

(i) Residual analysis

(ii) Multicolinearity

(iii)Non-linear regression analysis

(iv)Exponential smoothing

Formulate

λ = Rate of arrival of units

μ = Rate of service completion

θ = λ/μ

H = Number of working hours per day

$P_{(n)}$ = Probability of "n" units in the queuing system

L_s = Average number of units in queuing system

L_q = Average number of units in queue

W_s = Average time spent by units in queuing system

W_q = Average time spent by units in queue

$P(n) = \theta P(n-1)$ ———— (1)

$P(n) = \theta^n P(0)$ ———— (2)

$P(n) = \theta^n (1-\theta)$ ———— (3)

(Probability that queuing system empty) = $(1 - \theta)$ ———— (4)

(Probability that the server is idle) = $(1 - \theta)$ ———— (5)

(Number of hours server idle per day) = $H(1 - \theta)$ ———— (5)

$L_s = \theta/(1 - \theta)$ ———— (7)

$L_q = \theta^2/(1 - \theta)$ ———— (8)

$L_s = \lambda W_s$ ———— (9)

$L_q = \lambda W_q$ ———— (10)

(i) $EOQ = \sqrt{\frac{2DA}{C}}$ (without stock outs)

(ii) $K = \frac{DA}{Q} + \frac{1}{2}QC$ (without stock outs)

(iii) $EOQ = \sqrt{\frac{2DA}{C}\frac{(C+S)}{S}}$ (with stock outs)

(iv) $a = \frac{S \times EOQ}{(C+S)}$ (with stock outs)

Regression Analysis

$$r = \frac{\Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n}}{\sqrt{\left(\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right)\left(\Sigma y^2 - \frac{(\Sigma y)^2}{n}\right)}}$$

$$b = \frac{n\Sigma xy - (\Sigma x)(\Sigma y)}{n\Sigma x^2 - (\Sigma x)^2} \qquad a = \frac{\Sigma y}{n} - b\frac{\Sigma x}{n}$$

$$SSE = \Sigma y^2 - a\Sigma y - b\Sigma xy$$

$$SST = \Sigma y^2 - \frac{(\Sigma y)2}{n}$$

$$R^2 = \left(1 - \frac{SSE}{SST}\right)$$

$$S_{yx} = \sqrt{\frac{SSE}{n-2}}$$

$$SSX = \Sigma x^2 - \frac{(\Sigma x)\,2}{n}$$

$$S_b = \frac{S\,yx}{\sqrt{SSX}}$$