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The Open University of Sri Lanka
Department of Electrical and Computer Engineering
ECX5241 – Distributed Parameter Systems
Final Examination – 2014/2015

Date: 2015-09-17 Time:0930-1230

The paper contains two sections A and B. Answer **four** questions by answering **any three from section A** and **one question from section B**.

Section A

(20 Marks for each)

Q1.

(a) Compare "Distributed parameter systems" and "Lumped parameter systems"

(b) Choose any physical system and explain how it can be modeled as a distributed parameter system.

Q2.

(a) Verify the following vector identities where the vectors are defined in Cartesian coordinates,

i) $\nabla.(\boldsymbol{A} \times \boldsymbol{B}) = \boldsymbol{B}.\nabla \times \mathbf{A} - \mathbf{A}.\nabla \times \mathbf{B}$

ii) $\nabla \times (\nabla \times \boldsymbol{A}) = \nabla(\nabla.\boldsymbol{A}) - \nabla^2 \boldsymbol{A}$

(b) Let $\underline{\boldsymbol{E}} = (2xy + z^3)\boldsymbol{a}_x + x^2\boldsymbol{a}_y + 3z^2x\,\boldsymbol{a}_z$.

i) Show that $\boldsymbol{E}$ is a conservative vector field.

ii) Find the scalar potential of $\boldsymbol{E}$.

Q3.

(a) State the "divergence theorem".

(b) Let $\boldsymbol{r} = x\boldsymbol{a}_x + y\boldsymbol{a}_y + z\boldsymbol{a}_z$,

(i) Find the divergence and curl of the vector field: $\boldsymbol{E} = \frac{x}{|r|}\boldsymbol{a}_x$.

(ii) Prove that $\nabla.\boldsymbol{r} = 3$ and $\nabla \times \boldsymbol{r} = 0$.

Q4.

(a) Starting from the general form, explain how second order partial differential equations are classified as hyperbolic, parabolic and elliptic equations.

(b) Discuss and compare the properties of *the Wave equation, the Diffusion equation* and *the Laplace's equation.*

Section B

(40 Marks for each)

Q5.

Find the temperature in a thin metal rod of length L, with both ends insulated (so that there is no passage of heat through the ends) and with initial temperature in the rod $sin\left(\frac{\pi x}{L}\right)$.

(Hint: $\frac{\partial^2 \theta}{\partial x^2} = c^2 \frac{\partial^2 \theta}{\partial t^2}$).

Q6.

A tightly stretched string with fixed ends at $x = 0$ and $x = d$ is initially in equilibrium position. It is set vibrating by giving to each of its points an initial velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = a \sin\frac{\pi x}{d}$

Find the displacement $y(x, t)$.

(Hint: $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$)