The Open University of Sri Lanka
Faculty of Engineering Technology

Study Programme	: Bachelor of Technology Honours in Engineering
Name of the Examination	: Final Examination
Course Code and Title	: **DMX 6578 / MEX 6278 – FLUID MECHANICS**
Academic Year	: 2017/18
Date	: 16th, February 2019
Time	: 9.30am -12.30pm
Duration	: 3 hours

General Instructions

1. Read all instruction carefully before answering the questions.
2. This question paper consists of 8 questions. All questions carry equal marks.
3. Answer **any 5** questions only.

Q1.

(a). By considering an infinitesimally small cubical fluid element, show that the general equation of continuity in three-dimensional space can be expressed as, [4 marks]

$$\frac{\partial \rho}{\partial t} + \nabla.\left(\rho \vec{V}\right) = 0$$

where ,

$$\nabla = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \qquad \text{and} \qquad \vec{V} = u\underline{i} + v\underline{j} + w\underline{k}$$

(b). By using the above relationship, obtain the continuity equation for the following fluid flows. [4 marks]

I. 2D - Compressible flow

II. 3D- Steady - Incompressible flow

(c). The velocity components for **X** and **Y** directions of a 3D-steady incompressible flow are $u = x^2 + y^2 z^3$ and $v = -(xy + yz + zx)$ respectively. Find the missing component of the velocity distribution such that continuity equation is satisfied. [12 marks]

Q2.

(a). By considering the forces acting on the differential control volume of a fluid, Write down the expression for the following terms in usual notations.

I. Surface forces. [2 marks]

II. Body forces. [2 marks]

III. Material derivative. [2 marks]

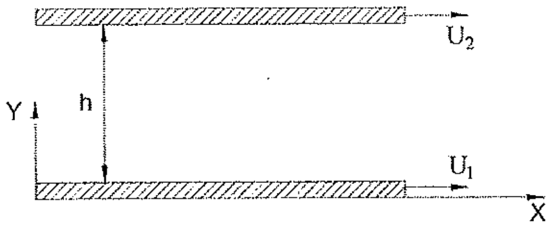

Figure Q2

(b). Show that the flow of a liquid per unit width (Q) for laminar flow between two parallel plates moving in the same direction with different velocities, as shown in Figure Q2, is given by [10 marks]

$$Q = \frac{h}{2}(U_1 + U_2) - \frac{1}{12\mu}h^3\frac{dp}{dx}$$

(c). Derive an expression for the shear stress on top plate. [4 marks]

Q3.

(a). Discuss the importance of Reynolds Number in relation to the behaviour of boundary layer. [2 marks]

(b). Explain the significance of the boundary layer displacement thickness δ^* and the momentum thickness θ. Express δ^* and θ as integrals of the boundary layer velocity profiles on a smooth flat plate. [4 marks]

(c). Calculate the ratio (δ^*/δ) for a laminar boundary layer with a velocity profile given by, [6 marks]

$$\frac{u}{U_\infty} = 2\frac{y}{\delta} - \frac{y^2}{\delta^2}$$

(d). Show that the frictional drag force per unit width (F_d) due to the boundary layer is given by, [8 marks]

$$F_d = \frac{2\delta}{15}\rho U_\infty^{\ 2}$$

Q4.

(a). Briefly explain an application of the potential flow theory. [3 marks]

(b). State the conditions of a fluid flow that are required for the existence of a stream function and a velocity potential function. [3 marks]

(c). A fluid flow field is given by,

$$V = \left(6xt + yz^2\right)i + \left(3t + xy^2\right)j + (xy - 2xyz - 6tz)k$$

[14 marks]

Prove that the flow is steady incompressible fluid flow. Calculate the velocity and acceleration at the point (2,2,2) at t=2.0 .

Q5.

(a). By using suitable examples, explain dimensional homogeneity, geometric similarity, kinematic similarity and dynamic similarity. [3 marks]

(b). State Buckingham π theorem. [3 marks]

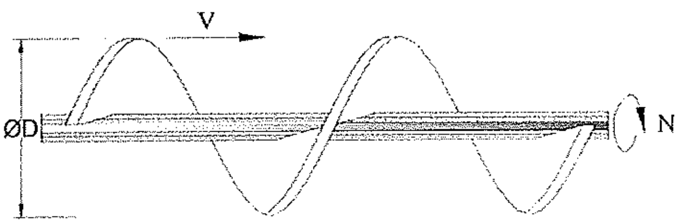

Figure Q5

(c). The thrust (T) of a screw propeller (as shown in Figure Q5) is depends on the diameter (D), speed of advance (V), revolution per second (N), fluid density (ρ) and the coefficient of viscosity (μ). Show that , [8 marks]

$$T = \rho D^2 V^2 \phi\left(\frac{VD\rho}{\mu}, \frac{DN}{V}\right)$$

[6 marks]

(d). The characteristics of a propeller of 4.8m diameter and rotational speed 120 rpm are examined by means of a geometrically similar model of 600mm diameter. When the model is run at 480 rpm by a torque of 30 Nm, the thrust developed is 300N and the speed of advance is $3ms^{-1}$. Determine the speed of advance (V) and Torque (τ) for the full scale propeller.

Efficiency (η) of a propeller is given by $\eta = \frac{Output}{Input} = \frac{TV}{\tau N}$

Q6.

(a). Write down the Bernoulli equation for 2D and state the assumptions that you have to make in order to apply it. 3 marks

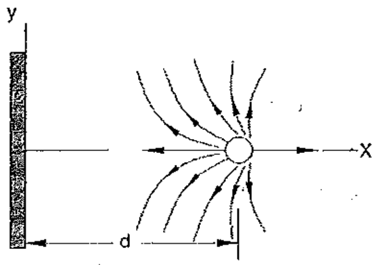

Figure Q6

(b). A source of strength **m** is located at a distance **d** from a vertical solid wall as shown in Figure 3. The velocity potential for this incompressible, irrotational flow is given by

$$\emptyset = \frac{m}{4\pi}\{ln[(x-d)^2 + y^2] + ln[(x+d)^2 + y^2]\}$$

I. Show that there is no flow through the wall. 5 marks

II. Determine the velocity distribution along the wall. 6 marks

III. Determine the pressure distribution along the wall, assuming $p=p_0$ far from the source. 6 marks

Q7.

(a). Explain the importance of Laplace equation in relation to the potential flow theory. 6 marks

(b). What is the physical significance of a Rankine oval of equal axes? 6 marks

(c). A uniform flow of velocity U=$\boldsymbol{10\ ms^{-1}}$ in the positive x-direction is flowing over a doublet of strength μ=$\boldsymbol{15\ m^2s^{-1}}$. The doublet is in the line of the uniform flow. The polar coordinates of a point **P** in the flow field are $\boldsymbol{0.9m}$ and $\boldsymbol{30^0}$. Determine,

I. Stream line function at point P 6 marks

II. Resultant velocity at point P. 8 marks

Stream function for a doublet in a uniform flow in usual notations is given by,

$$\psi = \left[Ur - \frac{\mu}{2\pi r}\right]\sin\theta$$

Q8.

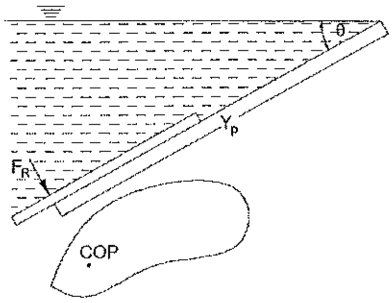

Figure Q8-a

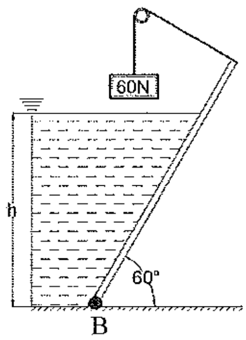

Figure Q8-b

a. By considering the given Figure Q8-a, show that distance, from the free surface, to the center of pressure on an inclined immersed plane surface area is given by, **8 marks**

$$y_p = \frac{Second\ moment\ of\ area}{First\ moment\ of\ area}$$

b. A 6m × 2m rectangular gate hinged at the base (B) is inclined at an angle of 60° to the horizontal. The upper end of the gate is kept in position by a weight of 60kN acting at angle of 90° as shown in Figure Q8-b. Neglecting the weight of the gate, compute the water level h for which the gate will start to fall. **12 marks**

Nervier-Stokes equations for incompressible flow:-

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=\rho g_x-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=\rho g_y-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=\rho g_z-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)$$

Frictional drag force per unit width due to laminar boundary layer :- $F_d = \rho\int_0^\delta u(U_\infty - u)dy$

Acceleration vector:- $\vec{a} = a_x i + a_y j + a_z k$ ***and*** $\vec{a}_x = \frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}$

Cartesian Coordinates:- $\nabla = \frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}$

Cylindrical Coordinates:- $\nabla = \frac{\partial}{\partial r}+\frac{1}{r}\frac{\partial}{\partial \theta}+\frac{\partial}{\partial z}$ $\quad V_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}$ $\quad V_\theta = -\frac{\partial \psi}{\partial r}$

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