



The Open University of Sri Lanka  
Faculty of Engineering Technology  
Department of Electrical and Computer Engineering

Study Programme	: Bachelor of Technology Honours in Engineering
Name of the Examination	: Final Examination
Course Code and Title	: <b>EEX6543, ECX6243 Microwave Engineering and Applications</b>
Academic Year	: 2017/18
Date	: 3 <sup>rd</sup> February 2019
Time	: 0930 -1230 hrs

### General Instructions

1. Read all instructions carefully before answering the questions.
  2. This question paper consists of **Eight (8)** questions in **Seven (6)** pages.
  3. Answer any **Five (5)** questions only. All questions carry equal marks.
  4. Answer for each question should commence from a new page.
  5. Relevant charts/ codes are provided.
  6. This is a Closed Book Test (CBT).
  7. Answers should be in clear hand writing.
  8. Do not use Red colour pen.
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1. Using the Maxwell's equations the time derivatives and space derivatives of electric field  $\vec{E}$  can be related using the following equation:

$$\nabla^2 \vec{E} = \mu \left( \sigma \frac{\partial \vec{E}}{\partial t} + \varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} \right)$$

- (a) Define the parameters  $\mu, \sigma$  and  $\varepsilon$ . [3 marks]  
 (b) If the electric field variation is sinusoidal ( $E = E_0 e^{j\omega t}$ ) show that the above equation reduces to the standard form:

$$\nabla^2 \vec{E} = \gamma^2 \vec{E}$$

Also find the value of  $\gamma$ . [6 marks]

- (c) If the medium of propagation is a perfect dielectric,  
 (i) find the value of  $\gamma$ . [3 marks]  
 (ii) show that the wave propagates without any attenuation. [4 marks]

Assume sinusoidal variation of the field components.

- (d) If the medium of wave propagation is free space, simplify the given equation. [4 marks]

2. (a) What are the possible modes of wave propagation in a rectangular waveguide? [2 marks]

- (b) Various field components inside a parallel plate waveguide are related through following equations:

$$H_x = \frac{j\omega\varepsilon}{k_c^2} \frac{\partial E_z}{\partial y} \quad E_x = -\frac{j\beta}{k_c^2} \frac{\partial E_z}{\partial x}$$

$$H_y = -\frac{j\omega\varepsilon}{k_c^2} \frac{\partial E_z}{\partial x} \quad E_y = -\frac{j\beta}{k_c^2} \frac{\partial E_z}{\partial y}$$

- (i) What is the mode of propagation of the wave? Justify your answer. [3 marks]  
 (ii) If  $E_z(x, y, z) = A_n \sin\left(\frac{n\pi y}{d}\right) e^{-j\beta z}$ , find the field components  $E_x, E_y, H_x$  and  $H_y$ . [7 marks]  
 (iii) Can TEM mode propagate inside  
 1. a parallel plate waveguide? [2 marks]

2. a rectangular waveguide?

[2 marks]

(c) Briefly explain why inner surface of a rectangular waveguide is coated with a metallic layer having a very high conductivity? [4 marks]

3. (a) A plane wave is propagating in a lossless dielectric medium has an electric field strength  $E_x = E_0 e^{j(\omega t - \gamma_0 z)}$

(i) Evaluate the wave number  $k$ ? [2 marks]

(ii) Find the wavelength  $\lambda$ . [3 marks]

(iii) Find the phase velocity of the wave. [3 marks]

(b) The propagation constant  $\gamma$  of a rectangular waveguide is given by

$$\gamma^2 = k_c^2 - k^2, \text{ where } k_c \text{ is the cutoff wave number.}$$

(i) Show that if  $k_c^2 < \omega^2 \mu \epsilon$ , an electromagnetic wave can propagate in the waveguide.  $\omega$  is the angular frequency of the wave.  $\epsilon$  and  $\mu$  are the permittivity and the permeability of the dielectric medium inside the waveguide respectively. [4 marks]

(ii) If  $k_c^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$ , where  $a$  and  $b$  breadth and the height of the waveguide respectively, derive an expression for the cutoff wavelength of the waveguide. [5 marks]

(iii) Use the answer of (ii) to find the cutoff wavelength for the dominant TE mode. [3 marks]

4. (a) The first few roots of  $J_n(x) = 0$  are shown in Table.1, where  $J_n(x)$  is a Bessel function of first kind and n-th order.

First 3 Roots of $J_n(x) = 0$			
$n$	$x_{n1}$	$x_{n2}$	$x_{n3}$
0	2.4	5.5	8.65
1	3.8	7.0	10.2
2	5.1	8.4	11.6

Table.1

For an air filled circular wave guide operating in the TM mode, the axial field strength  $E_z$  is given by  $E_z(\rho, \phi) = (A \sin n\phi + B \cos n\phi) J_n(\rho k_c)$ , where  $\rho$  and  $\phi$  are the radial- and the angular coordinates in the cylindrical coordinate system.  $A$  and  $B$  are constants. The internal radius of the cylindrical waveguide is  $a$ .

- (i) Using the boundary conditions show that  $J_n(k_c a) = 0$  [6 marks]
- (ii) If  $a = 4 \text{ cm}$ , find the cutoff wavelength and the cutoff frequency of the waveguide for  $TM_{01}$  mode. [4 marks]
- Assume that the phase velocity of an electromagnetic wave in air is  $3 \times 10^8 \text{ m/s}$ .

(b) A rectangular waveguide is operating in the TE mode. The electric field component in the  $x$ -direction is given by  $E_x = B \cos(k_x x) \sin(k_y y) e^{-j\beta z}$  where  $\beta$  is the propagating constant in the  $z$ -direction. Suppose the waveguide is converted into a resonator by shorting the waveguide at  $z = 0$  and  $z = c$ . Assuming that perfect reflection is taking place at  $z = c$ ,

- (i) derive an expression for the resultant electric field strength in the  $x$ -direction due to the reflection of the wave at  $z = c$ . [resultant field can be found by superimposing the incident wave with the reflected wave]. What is the reflection coefficient at  $z = c$ ? [6 marks]
- (ii) Using the result of (i) show that  $\beta = \frac{p\pi}{c}$ , where  $p$  is an integer. [4 marks]

5. (a)

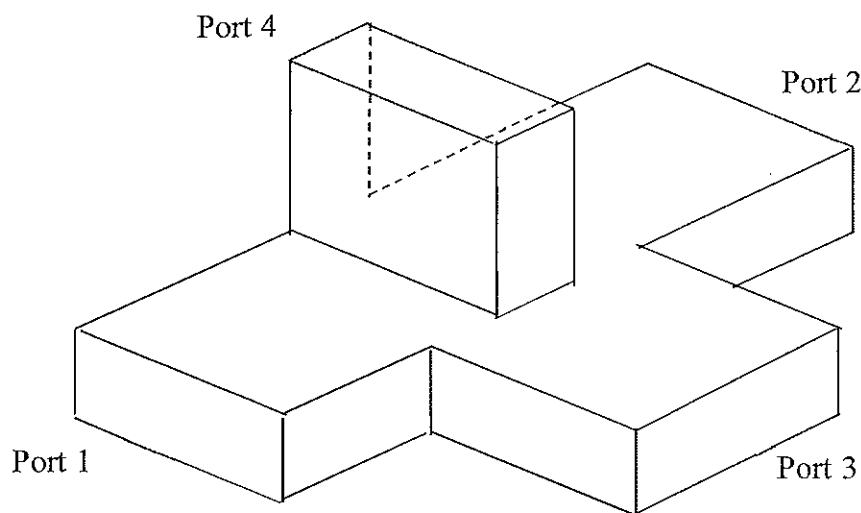


Fig. 5(a)

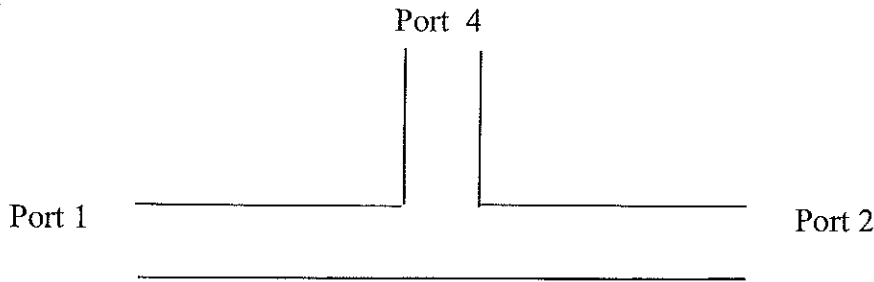


Fig. 5(b)

A magic Tee and its vertical section are shown in the Figs. 5(a) and 5(b).

- (i) If the Tee is excited at  $TE_{10}$  using a source connected to any of the above ports, indicate the direction of the E-field. [2 marks]
  - (ii) If the excitation is from Port 1 draw the E-field lines in other three ports. [3 marks]
  - (iii) If Port 3 is excited instead of Port 1 repeat (ii). [3 marks]
  - (iv) Consider the scattering matrix of the above device.
    1. What is the relationship between  $s_{14}$  and  $s_{24}$ ? Justify your answer. [2 marks]
    2. What is the relationship between  $s_{13}$  and  $s_{23}$ ? Justify your answer. [2 marks]
- (b) (i) Write an expression for directivity for a directional coupler defining all terms. [2 marks]
- (ii) State the necessary and sufficient conditions for a 4-port device to become a directional coupler. [2 marks]
- (c) Give one application of a directional coupler. Also explain the working principle of such a device. [4 marks]
6. (a) (i) With the help of a suitable diagram briefly explain the construction of a microstrip patch antenna. [3 marks]
- (ii) If the length  $L$  of a rectangular patch antenna is 10 cm., find the centre frequency  $f_c$  of the antenna. Assume that the relative permittivity  $\epsilon_r = 2$  for the antenna substrate and the speed of light in the free space is  $3 \times 10^8$  m/s. [3 marks]
- (iii) How can the bandwidth of the antenna be increased? [2 marks]
- (b) (i) What is frequency reuse? Why is it important for a cellular communication system? [3 marks]

(ii) The total bandwidth allocated for a particular communication system is 28 MHz. The system uses a frequency reuse factor of 7. The system provides duplex communication by allocating 50 kHz per channel.

1. Find the number of channels available per cell. [5 marks]
2. Find the maximum possible number of conversations that can take place in the system at a given instant if the system consists of 12 clusters. [4 marks]

7. (a) Briefly explain following terms:

- |                              |           |
|------------------------------|-----------|
| (i) cell splitting           | [3 marks] |
| (ii) co-channel interference | [3 marks] |
| (iii) sectoring              | [3 marks] |

Note: Also you should explain the influence of the above factors on the performance of the system.

(b) A cellular operator is operating in a certain area with 400 cells. Each cell is allocated 20 channels. The average call duration of a user is 4 minutes and each user makes 1 call per minute on average.

Find the total number of subscribers that can be supported by the system if the GOS (Grade Of Service) of the system is 3 %. [11 marks]

8. (a) A satellite is called geostationary if it appears to be stationary for an observer on earth.

- (i) Find the angular velocity of rotation of a geostationary satellite. [2 marks]
- (ii) If the above satellite is rotating in an orbit of 32000 km radius, find the instantaneous speed of the satellite. [2 marks]

(b) Briefly explain the principle of operation of a Traveling Wave Tube (TWT). [3 marks]

(c) Draw a block diagram of the uplink and down link of a satellite communication system. Briefly explain the function of each subsystem. [8 marks]

(d) Two rays emitted from a far away transmitter are incident on a parabolic reflector as shown in the Fig.8.

- (i) Write the relationship among AP, BQ, PF and QF. [2 marks]
- (ii) What happens if the above relationship does not exist? [3 marks]

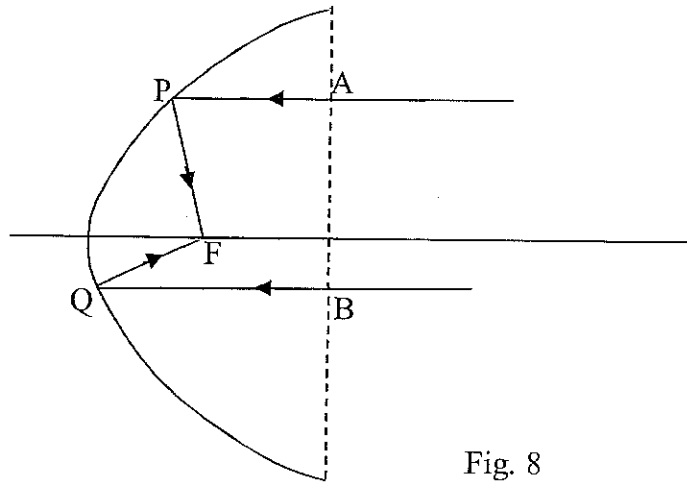


Fig. 8



## Erlang B Traffic Table

Maximum Offered Load Versus B and N

B is in %

N/B	0.01	0.05	0.1	0.5	1.0	2	5	10	15	20	30	40
1	.0001	.0005	.0010	.0050	.0101	.0204	.0526	.1111	.1765	.2500	.4286	.6667
2	.0142	.0321	.0458	.1054	.1526	.2235	.3813	.5954	.7962	1.000	1.449	2.000
3	.0868	.1517	.1938	.3490	.4555	.6022	.8994	1.271	1.603	1.930	2.633	3.480
4	.2347	.3624	.4393	.7012	.8694	1.092	1.525	2.045	2.501	2.945	3.891	5.021
5	.4520	.6486	.7621	1.132	1.361	1.657	2.219	2.881	3.454	4.010	5.189	6.596
6	.7282	.9957	1.146	1.622	1.909	2.276	2.960	3.758	4.445	5.109	6.514	8.191
7	1.054	1.392	1.579	2.158	2.501	2.935	3.738	4.666	5.461	6.230	7.856	9.800
8	1.422	1.830	2.051	2.730	3.128	3.627	4.543	5.597	6.498	7.369	9.213	11.42
9	1.826	2.302	2.558	3.333	3.783	4.345	5.370	6.546	7.551	8.522	10.58	13.05
10	2.260	2.803	3.092	3.961	4.461	5.084	6.216	7.511	8.616	9.685	11.95	14.68
11	2.722	3.329	3.651	4.610	5.160	5.842	7.076	8.487	9.691	10.86	13.33	16.31
12	3.207	3.878	4.231	5.279	5.876	6.615	7.950	9.474	10.78	12.04	14.72	17.95
13	3.713	4.447	4.831	5.964	6.607	7.402	8.835	10.47	11.87	13.22	16.11	19.60
14	4.239	5.032	5.446	6.663	7.352	8.200	9.730	11.47	12.97	14.41	17.50	21.24
15	4.781	5.634	6.077	7.376	8.108	9.010	10.63	12.48	14.07	15.61	18.90	22.89
16	5.339	6.250	6.722	8.100	8.875	9.828	11.54	13.50	15.18	16.81	20.30	24.54
17	5.911	6.878	7.378	8.834	9.652	10.66	12.46	14.52	16.29	18.01	21.70	26.19
18	6.496	7.519	8.046	9.578	10.44	11.49	13.39	15.55	17.41	19.22	23.10	27.84
19	7.093	8.170	8.724	10.33	11.23	12.33	14.32	16.58	18.53	20.42	24.51	29.50
20	7.701	8.831	9.412	11.09	12.03	13.18	15.25	17.61	19.65	21.64	25.92	31.15
21	8.319	9.501	10.11	11.86	12.84	14.04	16.19	18.65	20.77	22.85	27.33	32.81
22	8.946	10.18	10.81	12.64	13.65	14.90	17.13	19.69	21.90	24.06	28.74	34.46
23	9.583	10.87	11.52	13.42	14.47	15.76	18.08	20.74	23.03	25.28	30.15	36.12
24	10.23	11.56	12.24	14.20	15.30	16.63	19.03	21.78	24.16	26.50	31.56	37.78
25	10.88	12.26	12.97	15.00	16.13	17.51	19.99	22.83	25.30	27.72	32.97	39.44
26	11.54	12.97	13.70	15.80	16.96	18.38	20.94	23.89	26.43	28.94	34.39	41.10
27	12.21	13.69	14.44	16.60	17.80	19.27	21.90	24.94	27.57	30.16	35.80	42.76
28	12.88	14.41	15.18	17.41	18.64	20.15	22.87	26.00	28.71	31.39	37.21	44.41
29	13.56	15.13	15.93	18.22	19.49	21.04	23.83	27.05	29.85	32.61	38.63	46.07
30	14.25	15.86	16.68	19.03	20.34	21.93	24.80	28.11	31.00	33.84	40.05	47.74
31	14.94	16.60	17.44	19.85	21.19	22.83	25.77	29.17	32.14	35.07	41.46	49.40
32	15.63	17.34	18.21	20.68	22.05	23.73	26.75	30.24	33.28	36.30	42.88	51.06
33	16.34	18.09	18.97	21.51	22.91	24.63	27.72	31.30	34.43	37.52	44.30	52.72
34	17.04	18.84	19.74	22.34	23.77	25.53	28.70	32.37	35.58	38.75	45.72	54.38
35	17.75	19.59	20.52	23.17	24.64	26.44	29.68	33.43	36.72	39.99	47.14	56.04
36	18.47	20.35	21.30	24.01	25.51	27.34	30.66	34.50	37.87	41.22	48.56	57.70
37	19.19	21.11	22.08	24.85	26.38	28.25	31.64	35.57	39.02	42.45	49.98	59.37
38	19.91	21.87	22.86	25.69	27.25	29.17	32.62	36.64	40.17	43.68	51.40	61.03
39	20.64	22.64	23.65	26.53	28.13	30.08	33.61	37.72	41.32	44.91	52.82	62.69
40	21.37	23.41	24.44	27.38	29.01	31.00	34.60	38.79	42.48	46.15	54.24	64.35
41	22.11	24.19	25.24	28.23	29.89	31.92	35.58	39.86	43.63	47.38	55.66	66.02
42	22.85	24.97	26.04	29.09	30.77	32.84	36.57	40.94	44.78	48.62	57.08	67.68
43	23.59	25.75	26.84	29.94	31.66	33.76	37.57	42.01	45.94	49.85	58.50	69.34

44	24.33	26.53	27.64	30.80	32.54	34.68	38.56	43.09	47.09	51.09	59.92	71.01
45	25.08	27.32	28.45	31.66	33.43	35.61	39.55	44.17	48.25	52.32	61.35	72.67
46	25.83	28.11	29.26	32.52	34.32	36.53	40.55	45.24	49.40	53.56	62.77	74.33
47	26.59	28.90	30.07	33.38	35.22	37.46	41.54	46.32	50.56	54.80	64.19	76.00
48	27.34	29.70	30.88	34.25	36.11	38.39	42.54	47.40	51.71	56.03	65.61	77.66
49	28.10	30.49	31.69	35.11	37.00	39.32	43.53	48.48	52.87	57.27	67.04	79.32
50	28.87	31.29	32.51	35.98	37.90	40.26	44.53	49.56	54.03	58.51	68.46	80.99
51	29.63	32.09	33.33	36.85	38.80	41.19	45.53	50.64	55.19	59.75	69.88	82.65
52	30.40	32.90	34.15	37.72	39.70	42.12	46.53	51.73	56.35	60.99	71.31	84.32
53	31.17	33.70	34.98	38.60	40.60	43.06	47.53	52.81	57.50	62.22	72.73	85.98
54	31.94	34.51	35.80	39.47	41.51	44.00	48.54	53.89	58.66	63.46	74.15	87.65
55	32.72	35.32	36.63	40.35	42.41	44.94	49.54	54.98	59.82	64.70	75.58	89.31
56	33.49	36.13	37.46	41.23	43.32	45.88	50.54	56.06	60.98	65.94	77.00	90.97
57	34.27	36.95	38.29	42.11	44.22	46.82	51.55	57.14	62.14	67.18	78.43	92.64
58	35.05	37.76	39.12	42.99	45.13	47.76	52.55	58.23	63.31	68.42	79.85	94.30
59	35.84	38.58	39.96	43.87	46.04	48.70	53.56	59.32	64.47	69.66	81.27	95.97
60	36.62	39.40	40.80	44.76	46.95	49.64	54.57	60.40	65.63	70.90	82.70	97.63
61	37.41	40.22	41.63	45.64	47.86	50.59	55.57	61.49	66.79	72.14	84.12	99.30
62	38.20	41.05	42.47	46.53	48.77	51.53	56.58	62.58	67.95	73.38	85.55	101.0
63	38.99	41.87	43.31	47.42	49.69	52.48	57.59	63.66	69.11	74.63	86.97	102.6
64	39.78	42.70	44.16	48.31	50.60	53.43	58.60	64.75	70.28	75.87	88.40	104.3
65	40.58	43.52	45.00	49.20	51.52	54.38	59.61	65.84	71.44	77.11	89.82	106.0
66	41.38	44.35	45.85	50.09	52.44	55.33	60.62	66.93	72.60	78.35	91.25	107.6
67	42.17	45.18	46.69	50.98	53.35	56.28	61.63	68.02	73.77	79.59	92.67	109.3
68	42.97	46.02	47.54	51.87	54.27	57.23	62.64	69.11	74.93	80.83	94.10	111.0
69	43.77	46.85	48.39	52.77	55.19	58.18	63.65	70.20	76.09	82.08	95.52	112.6
70	44.58	47.68	49.24	53.66	56.11	59.13	64.67	71.29	77.26	83.32	96.95	114.3
71	45.38	48.52	50.09	54.56	57.03	60.08	65.68	72.38	78.42	84.56	98.37	116.0
72	46.19	49.36	50.94	55.46	57.96	61.04	66.69	73.47	79.59	85.80	99.80	117.6
73	47.00	50.20	51.80	56.35	58.88	61.99	67.71	74.56	80.75	87.05	101.2	119.3
74	47.81	51.04	52.65	57.25	59.80	62.95	68.72	75.65	81.92	88.29	102.7	120.9
75	48.62	51.88	53.51	58.15	60.73	63.90	69.74	76.74	83.08	89.53	104.1	122.6
76	49.43	52.72	54.37	59.05	61.65	64.86	70.75	77.83	84.25	90.78	105.5	124.3
77	50.24	53.56	55.23	59.96	62.58	65.81	71.77	78.93	85.41	92.02	106.9	125.9
78	51.05	54.41	56.09	60.86	63.51	66.77	72.79	80.02	86.58	93.26	108.4	127.6
79	51.87	55.25	56.95	61.76	64.43	67.73	73.80	81.11	87.74	94.51	109.8	129.3
80	52.69	56.10	57.81	62.67	65.36	68.69	74.82	82.20	88.91	95.75	111.2	130.9
81	53.51	56.95	58.67	63.57	66.29	69.65	75.84	83.30	90.08	96.99	112.6	132.6
82	54.33	57.80	59.54	64.48	67.22	70.61	76.86	84.39	91.24	98.24	114.1	134.3
83	55.15	58.65	60.40	65.39	68.15	71.57	77.87	85.48	92.41	99.48	115.5	135.9
84	55.97	59.50	61.27	66.29	69.08	72.53	78.89	86.58	93.58	100.7	116.9	137.6
85	56.79	60.35	62.14	67.20	70.02	73.49	79.91	87.67	94.74	102.0	118.3	139.3
86	57.62	61.21	63.00	68.11	70.95	74.45	80.93	88.77	95.91	103.2	119.8	140.9
87	58.44	62.06	63.87	69.02	71.88	75.42	81.95	89.86	97.08	104.5	121.2	142.6
88	59.27	62.92	64.74	69.93	72.82	76.38	82.97	90.96	98.25	105.7	122.6	144.3
89	60.10	63.77	65.61	70.84	73.75	77.34	83.99	92.05	99.41	107.0	124.0	145.9
90	60.92	64.63	66.48	71.76	74.68	78.31	85.01	93.15	100.6	108.2	125.5	147.6