(59)

00109

THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Mathematics & Philosophy of Engineering

Bachelor of Technology Honors in Engineering / Bachelor of Software Engineering Honors

Final Examination (2017/2018)
MHZ4340 /MHZ4360/ MPZ4140 /MPZ4160: Discrete Mathematics I

Date: 06th February 2019 (Wednesday) **Time: 9:30 am – 12:30 pm**

Instruction:

- **Answer only six questions.**
- **Please answer a total of six questions choosing at least one from each single section.**
- **All symbols are in standard notation and state any assumption that you made.**

SECTION – A

Q1.

I. Decide which of the following are propositions. What are the truth values of those that are proposition? **[20%]**

a) "$x > 3$" ;
b) "$\sqrt{2}$ is an irrational number" ;
c) "if $19 - 15 = 8$ then, $10 + 3 = 17$ or $6 + 9 = 15$";
d) "If x is an even integer, then x^2 is also even".

II. State the "convers", "inverse", and "contrapositive" of each of the following statement: **[30%]**

a) If robbery was the motive for the crime then the victim had money;
b) If the question papers were not easy, then we do not pass the examination.

III. Let p, q, and r be three statements.
Verify that $(p \longrightarrow q) \rightarrow [(p \wedge r) \rightarrow (q \wedge r)]$ is a tautology or not. **[20%]**

IV. Determine the truth value and Negation of the each of the following statements: **[20%]**

a) $\forall x \in \mathbb{R},\ |x| = x;$
b) $\forall m \in \mathbb{R},\ m < m + 2.$

V. Show that $\sim [p \vee (\sim p \wedge q)] \equiv \sim (p \vee q)$ using laws of the algebra of propositions, where $p, q,$ and r are propositions **[10%]**

Q2.

I. Test the validity of the following arguments:

a) If I work hard, then I will get a raise.
If I get a raise, then I will buy a boat.

Therefore If I don't buy a boat, then I must not have worked hard. **[25%]**

b) If there is cream, then I will drink coffee.
If there is a donut, then I will drink coffee.
There is no cream and there is a donut.

Therefore I drink coffee. **[25%]**

II. By using truth tables, prove Distribution laws of propositions. **[20%]**

III. Proof by contrapositive, show that "if n is an integer and $n^3 + 5$ is odd, then the n is even". **[30%]**

Q3.

I. Prove that all $m, n \in \mathbb{Z}$, if m, n are divisible by 3, then mn is divisible by 9. **[10%]**

II. Using Mathematical induction, for a positive integer n, prove each of the following: **[50%]**

a) $2 + 7 + 12 + \cdots . + (5n - 3) = \frac{n(5n-1)}{2}$ for all $n \geq 1$;

b) $n! > 2^n$ for all $n \geq 4$.

III. Prove directly that the sum of any two odd integers is an even integer. **[15%]**

IV. By giving a counter example, disprove each of the following statements:

a) $\forall p, q, x \in \mathbb{Z}$, if $pq = x$, then $p = \frac{x}{q}$. **[15%]**

b) For all positive integer $n, n^2 - 2n$ is positive **[10%]**

SECTION – B

Q4.

I. Write down the elements in each of the following set: **[20%]**

a) $A = \{x : x^3 - 16x = 0, and\ x \in \mathbb{Z}^-\}$;

b) $B = \{x : x < 13,\ x = 2n,\ n \in \mathbb{Z}^+\}$;

c) $C = \{x : x = n^3 + n^2,\ 0 \leq n \leq 5, n \in \mathbb{Z}\}$,

d) $D = \{x : x \in \mathbb{Z}^+, x \text{ is odd}\}$.

II. Let $P = \{x: x \in \mathbb{N}\}$, $Q = \{x: x \text{ is a prime number}, x \leq 10\}$, and $R = \{1,3,5,7,9\}$. Find **[15%]**

a) $P \oplus Q$;

b) $Q \oplus R$;

c) $P \cap (Q \oplus R)$, where $\oplus$ is symmetric difference.

III.

a) Define the Cartesian product of set A and B. **[05%]**

b) $M = \{3, 33, 333\}$ and $N = \{2, \ 22, \ 222\}$. Find $M \times N$ and N^2. **[20%]**

IV. Let $S = \{1, 2, \ \{1, 2\}, 12\}$. Find the power set $P(S)$ of S. **[10%]**

V. Without using Venn diagram, Show that

$A \cap (B \oplus C) = (A \cap B) \oplus (A \cap C)$, where $\oplus$ is symmetric difference. **[30%]**

Q5.

I. Let $\forall x \in \{1,2,3,4,5\}$, $f(x) = x^2$ and $\forall x \in \{2,3,4,5,6,,9\}$, $g(x) = x - 1$.

a) Write down the domains of $f \circ g$ and $g \circ f$, **[15%]**

b) Find functions of $f \circ g$ and $g \circ f$, **[15%]**

c) Write down the images of $f \circ g$ and $g \circ f$. **[10%]**

II. Let $h: \mathbb{R}_0^- \to \mathbb{R}_0^+$ be a function defend by $h(x) = x^2 + 1$ for all $x \in \mathbb{R}_0^-$.

a) Show that $h(x)$ is a one to one function. **[10%]**

b) Find the inverse function $h^{-1}(x)$ of $h(x)$, if it exists. **[20%]**

III. Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{3/2\}$. Define $(x) = \frac{3x+5}{2x-6}$. Prove that $k(x)$ is invertible and find a formula for $k^{-1}(x)$. **[30%]**

Q6.

I. Let $A = \{2,5,6,8,12\}$ and $B = \{3,5,7,9,11\}$. Find the following relations from A to B.

a) $l_1 = \{(x,y) | \, x \leq y; \ x \in A, y \in B\}$ **[10%]**

b) $l_2 = \{(x,y) | \, x + 1 < y; x \in A, y \in B\}$. **[10%]**

II.

a) Define the equivalence relation by the usual notation. **[10%]**

b) Determine whether the following relations are equivalence relation or not.

α) If R_1 be the relation which is defined by aR_1b iff $a - b$ is an integer on the set $\mathbb{R}$ of real numbers. **[25%]**

β) If R_2 be the relation which is defined by aR_2b iff $a - b$ is an integer on the set $\mathbb{Z}_0^+$ of positive integers. **[25%]**

III. Show that "x is a factor of y" is a partial order relation, where $x, y \in \mathbb{Z}$. **[20%]**

SECTION – C

Q7.

I. Let $a, b,$ and c be any integer numbers. Prove that, **[30%]**

a) if $a|b$ and $c|d$, then $ac|bd,$

b) if $a|b$, $a > 0$ and $b > 0$, then $a \leq b,$

c) If $c|a$ and $c|b$, then $c|(3a - 5b),$

II. Let $x \in \mathbb{Z}$. If $(x - 1)|(x^2 - 3x + 5)$, then show that $(x - 1)|(2x^3 - 3x^2 + 4x)$. **[20%]**

III.

a) Define a prime number. **[05%]**

b) Let $a, b \in \mathbb{Z}^+$. Prove that if $b|a$ and $b|(a + 2)$, then $b = 1$ or 2 **[20%]**

c) If $n \geq 5$ is a prime number, show that $n^2 + 2$ is not prime number. **[25%]**

Q8.

I. Let a, b and c be integers. Show that $gcd(a, b) = gcd(a + cb, b)$. **[10%]**

II. Show that if $a, b,$ and c are positive integers such that $gcd(a, b) = 1$ and $a|bc$, then $a|c$. **[15%]**

III. Show that if a and b are relatively prime numbers, then $\gcd(a + 2b, 2a + b) = 1$or 3. **[30%]**

IV. Use the Euclidean algorithm to find the greatest common divisor of 4147 and 10672 and express it in terms of the two integers. **[25%]**

V. Either find all solutions or prove that there are no solutions for the Diophantine equation $2x + 13y = 31$. **[20%]**

Q9.

I Let a, b, c and d denote integers. Let m be a positive integers. Show that:

a) If $a \equiv b(mod\ m)$ and $c =\equiv d(mod\ m)$, then $a - c \equiv (b - d)(modm)$. **[10%]**

b) $a \equiv b(mod\ m)$, $b \equiv a(mod\ m)$ and $a - b \equiv 0(mod\ m)$ are equivalent statements. **[15%]**

c) If $ac \equiv bc(mod\ m)$ and $d \equiv gcd(m, c)$, then $a \equiv b(mod\ m/d)$. **[20%]**

II Solve the following system of congruence: **[55%]**

$$17x \equiv 3(mod\ 2)$$

$$17x \equiv 3(mod\ 3)$$

$$17x \equiv 3(mod\ 5)$$

$$17x \equiv 3(mod\ 7)$$