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The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Mathematics and Philosophy of Engineering

Study Programme	: Bachelor of Technology Honours in Engineering
Name of the Examination	: Final Examination
Course Code and Title	**: MHZ4530/MPZ4230 Engineering Mathematics II**
Academic Year	: 2017/18
Date	:01st February 2019
Time	: 0930-1230 hrs
Duration	**: 3 hours**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of Nine **(9)** questions in **Six (6)** pages.
3. Answer any **Six (6)** questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Relevant Statistic Tables and equations are provided.
6. This is a Closed Book Test (CBT).
7. Answers should be in clear hand writing.
8. Do not use Red colour pen.

Q1.

a) A tree trunk may be considered as a circular cylinder. Suppose that diameter of the trunk increases 1 inch per year and the height of the trunk increase 6 inchers per year. It is also founded that the diameter of the trunk is 5 inches when the height of the trunk is 100 inches. How fast the volume of the wood in the trunk increase? [25%]

b) Approximate $\sqrt{(3.012)^2 + (3.997)^2}$ using Taylor polynomial method. [25%]

c) Consider the vector force $\underline{F}\,(x,y) = (4xy - 3x^2z^2)\underline{i} + 2x^2\,\underline{j} - 2x^3z\underline{k}$

i. Show that the force $\underline{F}$ represent conservative field. [10%]

ii. Find a scalar potential φ such that $\underline{F} = \nabla\varphi$. [25%]

iii. Find the work done in moving a particle of unit mass under this field of force from the point $(0,0,0)$ to the point $(1,1,1)$. [15%]

Q2.

a) State the necessary condition for the function $f(z) = u(r,\theta) + iv(r,\theta)$ to be analytic. [10%]

b) Show that $v(r,\theta) = r^2\cos 2\theta - r\cos\theta$ is a harmonic function. [20%]

c). Determine the analytic function $f(z) = u(r,\theta) + i\,v(r,\theta)$, whose imaginary part is $v(r,\theta) = r^2\cos 2\theta - r\cos\theta$. [50%]

d) Express $f(z)$ in terms of z,where $z = re^{i\theta}$. [20%]

Q3. The study was conducted to identify the updating of ozone levels in California's South Coast Air Basin for the years $1981 - 1991$. It believes that the **number of days** that the ozone levels exceeded $0.2ppm$ depends on the **seasonal meteorological index**, which is the seasonal average 850 –millibar temperature. The following table gives the data.

Year	Days	Index
1981	91	18.7
1982	75	17.8
1983	106	18.2
1984	108	18.1
1985	88	18.3
1986	91	18.2
1987	48	17.1
1988	61	18.2
1989	43	17.3
1990	33	17.5
1991	36	16.6

a) Identify the independent variable and dependent variable. [10%]

b) Construct scatter plot for this data and identify the relationship type. [10%]

c) Confirm the relationship type by correlation coefficient value. [10%]

d) Write the best fit line for the given data. [20%]

e) Predict the number of days if the meteorological index is 16.8. [10%]

f) Following table represents some summary statistics for simple linear regression model from the above data. Fill in the blanks of the following table. [30%]

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	F_0
Regression	4967.2	1	4967.2	15.888
Residual	2813.7	... ...	... ...	
Total	... ...	10		

g) Test the significance of the regression model at 5% level of significance using above table. [10%]

Q4.

a) The average life span of $n = 100$ deceased persons was $\bar{x} = 71.8$ years. According to earlier studies, the population standard deviation is assumed to be $\sigma = 8.9$ years. According to this information, could it be concluded that the average life span μ of the population is greater than 70 years? The life span is supposed to be normally distributed.

i. Test the relevant hypothesis at 5% significance level. [20%]

ii. Construct confidence interval at 1% significance level. [20%]

b) A wine producer is considering using duo corks in place of full natural wood corks to reduce costs, but he concerned that it could affect buyer's perception of the quality of the wine. The wine producer shipped eight pairs of bottles of its best young wines to eight wine experts. Each pair includes one bottle with a natural wood cork and one with a duo cork. The experts are asked to rate the wines on a one to ten scale, higher numbers corresponding to higher quality. The results are:

Wine expert	Duo Cork	Wood cork
1	8.5	8.5
2	8	8.5
3	6.5	8
4	7.5	8.5
5	8	7.5
6	8	8
7	9	9
8	7	7.5

i. Give a point estimate for the difference between the mean ratings of the wine when bottled are sealed with different kinds of corks. [20%]

ii. Test, at the 10% level of significance, the hypothesis that on the average rating of duo corks remains same rating of the wine. [40%]

Q5.

a) Consider the differential equation $\frac{dy}{dx} = 1 - y$.

i. Using Euler's method find $y(0.1)$ and $y(0.2)$ considering step size 0.1 with initial condition $y(0) = 1$. [20%]

ii. Find the actual value of $y(0.1)$ and $y(0.2)$. [20%]

b) Applying the Runge Kutta Fourth order method for system of simultaneous differential equations $\frac{dy}{dx} = x + z$ and $\frac{dz}{dx} = x - y^2$ subject to the initial conditions $y(0) = 2$ and $z(0) = 1$, evaluate $y(0.1)$ and $z(0.1)$. [60%]

Note that: The forth order Runge Kutta method for the function $f(x_m, y_m)$is given bellow in the usual notation.

$$k_1 = hf(x_m, y_m)$$

$$k_2 = hf\left(x_m + \frac{1}{2}h, y_m + \frac{1}{2}k_1\right)$$

$$k_3 = hf\left(x_m + \frac{1}{2}h, y_m + \frac{1}{2}k_2\right)$$

$$k_4 = hf(x_m + h, y_m + k_3)$$

$$y_{m+1} = y_m + \frac{1}{6}(k_1+2k_2+2k_3+k_4)$$

Q6.

a) Classify the elliptic, parabolic and hyperbolic form of the second order partial differential equation. [10%]

b) Consider the parabolic partial differential equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ and h is the step size in the $x-$direction and k is the step size in the $t-$direction.

i. Show that the explicit formula for numerical solution of parabolic partial differential is $u_{i,j+1} = ru_{i-1,j} + (1-2r)u_{i,j} + ru_{i+1,j}$, where $r = \frac{k}{h^2}$. [20%]

ii. Show that the implicit formula for numerical solution of parabolic partial differential is $u_{i,j+1} = u_{i,j} - r[2u_{i,j+1} - u_{i+1,j+1} - u_{i-1,j+1}]$, where $r = \frac{k}{h^2}$. [20%]

c) Consider the heat equation

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = 0 \qquad 0 \le x \le \pi, \qquad 0 \le t$$

with boundary condition

$$u(0,t) = u(\pi,t) = 0, \quad 0 \le t$$

and initial conditions

$$u(x,0) = \sin x, \qquad 0 \le x \le \pi$$

Find solution for five steps of t explicit method with $h = 0.25\,\pi$. [50%]

Q7. Consider the series RLC circuit, inductance $(L) = 50$ mH , resistance $(R) = 150\ \Omega$, capacitance $(C) = 10\ \mu$F and voltage $(E(t)) = 100 \sin 1500t\ V$. Consider initial current and charge are zero.

a) Find the charge and current at time t of RLC circuit. [90%]

b) Indicate steady state solution of charge. [10%]

You may assume that,

Kirchhoff's voltage law for series circuit yield

$$V_L + V_R + V_C = E(t), \quad \text{where}$$

$$V_L = L\frac{di}{dt}, \quad V_R = iR, \quad V_C = \frac{1}{C}\int i\,dt, \quad i = \frac{dq}{dt}$$

Q8.

a) Consider the vectors $\underline{v_1} = (1,1,1), \underline{v_2} = (1,2,1)$ and $\underline{v_3} = (1,3,2)$.

   i. Determine whether the given vectors are linear independent or dependent. [10%]

   ii. Write down the vector $\underline{v} = (x_1, x_2, x_3)$ as a linear combination of $\underline{v_1}, \underline{v_2}$ and $\underline{v_3}$. [15%]

b) Consider the basis $S = (\underline{v_1}, \underline{v_2}, \underline{v_3})$ for $V_3(\mathbb{R})$, where $\underline{v_1} = (1,1,1), \underline{v_2} = (1,2,1)$ and $\underline{v_3} = (1,3,2)$. Let $T : V_3(\mathbb{R}) \to V_3(\mathbb{R})$ be a linear transformation for which $T(\underline{v_1}) = (3,4,5)$, $T(\underline{v_2}) = (4,5,7)$ and $T(\underline{v_3}) = (4,6,6)$.

   i. Find the formula for $T(x_1, x_2, x_3)$. [25%]

   ii. Find $T(1,2,3)$. [10%]

c) Consider the following linear transformation $L : \mathbb{R}^3 \to \mathbb{R}^3$,

   $L(x, y, z) = (x + y + z, 2x + 2y + 2z, 3x + 3y + 3z)$

   i. Find bases of the image and kernel of L, and hence determine the rank and nullity of linear transformation L. [30%]

   ii. Verify that the theorem $\dim[\ker L] + \dim[\text{range } L] = \dim L$. [10%]

Q9.

a) Let $Q(x_1, x_2, x_3) = {x_1}^2 + {x_2}^2 + {x_3}^2 - 2x_1x_2 - 2x_2x_3 - 2x_1x_3$ be a quadratic function of x_1, x_2 and x_3. Determine the symmetric matrix A corresponding to the above quadratic form. [10%]

b) Find the eigen values of A and its corresponding eigen vectors. [30%]

c) Find an orthogonal matrix P such that $D = P^{-1}AP$, where D is a diagonal matrix. [25%]

d) Using above results find D^n and D^{-1}, where n is a positive integer. [15%]

e) Derive the quadratic functions of $Q(\underline{x}) = \underline{x}^T A \underline{x}$ and $Q(\underline{y}) = \underline{y}^T D \underline{y}$

   where $\underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ and $\underline{y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$ [10%]

f) Show that $P^{-1} = P^T$, and hence show that $Q(\underline{x}) = Q(\underline{y})$, where $\underline{y} = P^T\underline{x}$. [10%]

-END-

Correlation coefficient value.

$$r = \frac{\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \overline{x})^2 \quad \sum_{i=1}^{n}(y_i - \overline{y})^2}}$$

Parameters of the fitted model

$$\widehat{\beta}_1 = \frac{\sum_{i=1}^{n} y_i x_i - \frac{(\sum_{i=1}^{n} y_i)(\sum_{i=1}^{n} x_i)}{n}}{\sum_{i=1}^{n} x_i^2 - \frac{(\sum_{i=1}^{n} x_i)^2}{n}}$$

$$\widehat{\beta}_0 = \overline{y} - \widehat{\beta}_1 \overline{x}$$

The best fitted line

$$\widehat{y} = \widehat{\beta}_0 + \widehat{\beta}_1 x$$

Source of Variation	**Sum of Squares**	**Degrees of Freedom**	**Mean Squares**	F_0
Regression	$SS_R = \hat{\beta}_1 S_{xy}$	1	MS_R	MS_R/MS_{Res}
Residual	$SS_{Res} = SS_T - \hat{\beta}_1 S_{xy}$	n-2	MS_{Res}	
Total	SS_T	n-1		

Test statistics when data are normally distributed

$$Z = \frac{\overline{x} - \mu_0}{\sigma/\sqrt{n}}$$

Point estimate

$$\overline{d} = \frac{\sum_{i=1}^{n} d_i}{n}$$

Sample standard deviation

$$S_D = \sqrt{\frac{\sum_{i=1}^{n}(d_i - \overline{d})^2}{n-1}}$$

Probabilities for the standard normal distribution

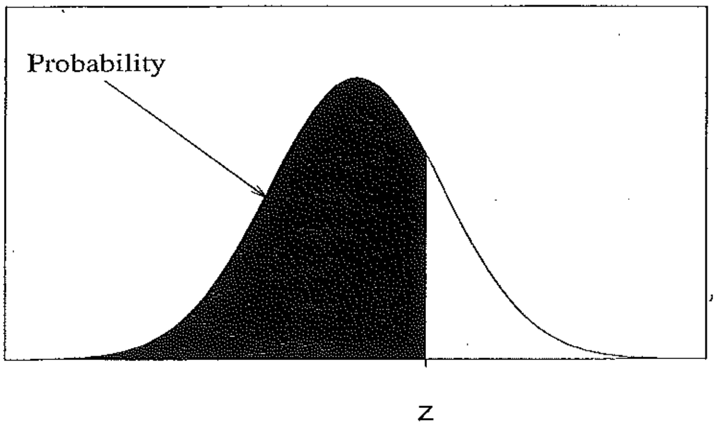
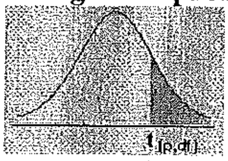


Table entry for *z* is the probability lying to the left of *z*

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

t table with right tail probabilities



df\p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	4.3178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405
15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728
16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.0150
17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216
19	0.256923	0.687621	1.327728	1.729133	2.09302	2.53948	2.86093	3.8834
20	0.256743	0.686954	1.325341	1.724718	2.08596	2.52798	2.84534	3.8495
21	0.256580	0.686352	1.323188	1.720743	2.07961	2.51765	2.83136	3.8193
22	0.256432	0.685805	1.321237	1.717144	2.07387	2.50832	2.81876	3.7921
23	0.256297	0.685306	1.319460	1.713872	2.06866	2.49987	2.80734	3.7676
24	0.256173	0.684850	1.317836	1.710882	2.06390	2.49216	2.79694	3.7454
25	0.256060	0.684430	1.316345	1.708141	2.05954	2.48511	2.78744	3.7251
26	0.255955	0.684043	1.314972	1.705618	2.05553	2.47863	2.77871	3.7066
27	0.255858	0.683685	1.313703	1.703288	2.05183	2.47266	2.77068	3.6896
28	0.255768	0.683353	1.312527	1.701131	2.04841	2.46714	2.76326	3.6739
29	0.255684	0.683044	1.311434	1.699127	2.04523	2.46202	2.75639	3.6594
30	0.255605	0.682756	1.310415	1.697261	2.04227	2.45726	2.75000	3.6460
inf	0.253347	0.674490	1.281552	1.644854	1.95996	2.32635	2.57583	3.2905

F table at 5% significance level

df2/df1	1	2	3	4	5	6	7	8	9	10
1	161.4476	199.5000	215.7073	224.5832	230.1619	233.9860	236.7684	238.8827	240.5433	241.8817
2	18.5128	19.0000	19.1643	19.2468	19.2964	19.3295	19.3532	19.3710	19.3848	19.3959
3	10.1280	9.5521	9.2766	9.1172	9.0135	8.9406	8.8867	8.8452	8.8123	8.7855
4	7.7086	6.9443	6.5914	6.3882	6.2561	6.1631	6.0942	6.0410	5.9988	5.9644
5	6.6079	5.7861	5.4095	5.1922	5.0503	4.9503	4.8759	4.8183	4.7725	4.7351
6	5.9874	5.1433	4.7571	4.5337	4.3874	4.2839	4.2067	4.1468	4.0990	4.0600
7	5.5914	4.7374	4.3468	4.1203	3.9715	3.8660	3.7870	3.7257	3.6767	3.6365
8	5.3177	4.4590	4.0662	3.8379	3.6875	3.5806	3.5005	3.4381	3.3881	3.3472
9	5.1174	4.2565	3.8625	3.6331	3.4817	3.3738	3.2927	3.2296	3.1789	3.1373
10	4.9646	4.1028	3.7083	3.4780	3.3258	3.2172	3.1355	3.0717	3.0204	2.9782