**THE OPEN UNIVERSITY OF SRI LANKA**
**Faculty of Engineering Technology**
**Department of Mathematics & Philosophy of Engineering**

**Bachelor of Technology Honors in Engineering**

**Final Examination (2017/2018)**
**MHZ3332/MPZ3132: Engineering Mathematics 1B**

**Date: 21st January 2019 (Monday)** **Time: 9:30 am – 12:30 am**

**Instruction:**

- **Answer only five questions.**
- **Number of pages in the paper – 05.**
- **All symbols are in standard notation and state any assumption that you made.**

**Important integrals**

- $\int e^{ax} sinbxdx = \frac{e^{ax}}{a^2+b^2}(asinbx - bcosbx)$
- $\int e^{ax} cosbxdx = \frac{e^{ax}}{a^2+b^2}(acosbx + bsinbx)$
- $\int f(x)e^{ax}\,dx = \frac{1}{a}f(x)e^{ax} - \frac{1}{a^2}f^1(x)e^{ax} + \frac{1}{a^3}f^2(x)e^{ax} \ldots\ldots$ [Sign alternate $(+ - + - + \cdots)$ ]
- $\int f(x)cosax\,dx = \frac{1}{a}f(x)sinax + \frac{1}{a^2}f^1(x)cosax - \frac{1}{a^3}f^2(x)sinax - \cdots$ [Sign alternate in pairs $- - + + - - + + - - \cdots$)]
- $\int f(x)sinax\,dx = -\frac{1}{a}f(x)cosax + \frac{1}{a^2}f^1(x)sinax + \frac{1}{a^3}f^2(x)cosax - \cdots$ [Sign alternate in pairs after the first term $(+ + - - + + - - + + - - \cdots)$]
- $\int secax\,dx = \frac{1}{a}ln|secax + tanax|$

**Q1.**

I. Write down the Fourier series expansion for the function $f(x)$ with period $2\pi$. **[10%]**

II. Find the Fourier series expansion for $f(x) = 30x^2 + 20x + 10$, where $-\pi < x < \pi$, and for the other values of $x$, $f(x) = f(x + 2k\pi)$, where $k \in \mathbb{Z}$.

Hence, find the value of $\sum_{n=1}^{\infty} \frac{1}{n^2}$. **[50%]**

III. Find Fourier series for the function $f(x)$, defined on $[-2, 2]$, where

$$f(x) = \begin{cases} -1, & -2 \le x \le 0, \\ 2, & 0 < x \le 2. \end{cases}$$ [40%]

**Q2.**

I. Define the Taylor series of $f(x)$ about $x = 0$.
Show that the Taylor series of the following functions about $x = 0$: [30%]

a) $\sin x = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!}$.

b) $\cos x = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!}$.

II. Using part I, find the Taylor series of $sin^2(x)$ and $cos^2(x)$ [20%]

III. Evaluate $\sin x$ at $\frac{\pi}{6}$ for the first 3 terms. [20%]

IV. Find the Taylor series of $g(x) = x^6 + 3\sin 2x$ at $x = 2$. [30%]

**Q3.**

I. Solve each of the following differential equation: [45%]

a) $(D^2 + 11D + 24)y = (e^{3x} + 1)^2$;

b) $(D^2 - 2D + 1)y = xe^x$;

c) $(D^2 + 25)y = \cos 4x$.

II. The differential equation:
$(D^2 - 4D - 12)y = x + \sin x$,
has a trial function of the form $y = A + bx + C\cos x + D\sin x$.
Write down the particular solution and general solution of the differential equation. [30%]

III. Using the result.

$$\frac{1}{(D+\alpha)} f(x) = e^{-\alpha x}\frac{1}{D} e^{\alpha x} f(x),$$

Find the general solution of the following differential equation:

$(D^2 - 5D + 6)y = 500\sin 4x$. [25%]

**Q4.**

I. Define the "Laplace transformation" of $f(t)$. **[05%]**
Find the "Laplace transformation" of each the following function: **[40%]**

a) $5e^{-7t} + 3\sin 2t - 9$;

b) $e^{3t}\cos 6t - t\cosh 3t$;

c) $(t + 3e^{-5t} + 6\cos 2t)\sin 2t$.

II. Find the inverse "Laplace transformation" of the following: **[30%]**

a) $\frac{19}{s+2} + \frac{1}{3s-5} - \frac{6}{s^2+64}$;

b) $\frac{6s-5}{s^2+7}$;

c) $\frac{s+7}{s^2-3s-10}$.

III. Using the "Laplace transformation", solve the following differential equation: **[25%]**

$$y'' - 3y' - 10y = 5e^{3t}\ ;\ y(0) = y'(0) = 0\,.$$

**Q5.**

I. The Probability density function of a discrete random variable is given by

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $P(X = x)$ | $0 \cdot 1$ | $3\lambda$ | $2\lambda$ | $0 \cdot 3$ | $0 \cdot 1$ |

Find

a) The value of $\lambda$, **[10%]**
b) $E(X)$, **[10%]**
c) $Var(X)$, **[10%]**
d) $E(4X^2 + 3X - 1)$. **[10%]**

II.

a) Define the Binomial distribution with usual notation. **[10%]**

b) Suppose there are 12 multiple choice questions in Computer class quiz. Each question has 5 possible answers, and only one of them is correct. Find the probability of having four or less correct answers if a student attempts to answer every question at random. **[25%]**

III. Assume that the GRE scores of a scholarship fits a normal distribution. The top 5% of applications (as measured by GRE scores) will receive scholarships.
If $GRE \sim N(500, 100^2)$, then how high does your GRE score have to be to qualify for a scholarship? **[25%]**

**Q6.**

I. Shade the complex region satisfying each of the following inequality: **[40%]**

a) $|z-3| < |z-5|$;

b) $|z-3i| \geq |Z+4|$

c) $|z+1| \geq |z-5|, 0 \leq \arg(z-1) \leq \frac{\pi}{3}$, and $|z-4| \leq 4$.

II. Express $Log(1+i)$ in the form of $x+iy$, where $x$ and $y$ are real numbers. **[20%]**

III. A particle moves along the curve $x = t^4 + 2,\ y = t^2,\ z = 3t + 8$ where $t$ is the time. Find the components of its velocity and acceleration at the time $t = 1$ in the direction of $8\boldsymbol{i} - 4\boldsymbol{j} + \boldsymbol{k}$. **[20%]**

IV. If $\underline{\boldsymbol{a}}$ and $\underline{\boldsymbol{b}}$ constant vectors, find $\frac{d}{dt} | 2t\underline{\boldsymbol{a}} + 3t^2\underline{\boldsymbol{b}} |$. **[20%]**

**Q7.**

I. Find the moment of inertia of the following bodies about the given axis: **[45%]**

a) A uniform circular ring mass $m_1$ and radius $a$, the axis perpendicular to the plane of the ring passing through the center.

b) A uniform hollow cylinder mass $m_2$, height $h$ and radius $a$ about its axis.

c) A uniform hollow sphere mass $m_3$ and radius $b$ the axis about diameter of sphere.

II. Find the moments of inertias above part (a), the axis perpendicular to the plane of the ring passing through the tangent line. **[10%]**

III.

a) Find the moment of inertia of the uniform rod $AB$ of mass $M_1$ and length $3l$, the axis perpendicular to the rod passing through the midpoint of the rod. **[15%]**

b) Above $AB$ rod, fixed at $B$ to a point on the circumference of a uniform hollow sphere of mass $M_2$ and radius $l$. The disc and the line of the rod lie in the same vertical plane with the center of the disc lying on the rod produced. Find the moment of inertia of the system about the axis perpendicular to the plane of the disc passing through $A$. **[30%]**

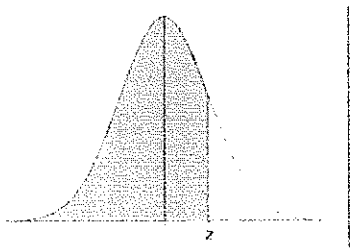
**Q8.**

I. A uniform square lamina $ABCD$ is immersed vertically in a homogeneous liquid of density $\rho$ with its plane vertical and $AB$ in the free surface. Find the liquid thrust on the lamina and the center of pressure. **[30%]**

A square hole $A'B'C'D'$ of side $b$ $(< a)$ is cut in the lamina. $ABCD$, $A'B'C'D'$ have the same center and their corresponding side are parallel. Find the thrust on the perforated lamina and show that the center of pressure of the perforated lamina is at the distance $\frac{4a^4-3a^2b^2-b^4}{6a(a^2-b^2)}$ from the free liquid surface. **[40%]**

II. A uniform rectangular lamina $PQRS$ is immersed vertically in a homogeneous liquid such that $PQ$ and $RS$ are horizontal. The distance to $PQ$ and $RS$ from the surface of the liquid are $l_1$ and $l_2$ respectively. Prove that the center of pressure of the lamina is at the distance $\frac{2}{3}\left(\frac{l_1^2+l_1l_2+l_2^2}{l_1+l_2}\right)$ from the free liquid surface. **[30%]**

# Standard Normal Cumulative Probability Table




**Cumulative probabilities for POSITIVE z-values are shown in the following table:**

| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| 3.4 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |