(29) 00749

# The Open University of Sri Lanka
# Faculty of Engineering Technology
# Department of Mathematics and Philosophy of Engineering

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| Study Programme | : Bachelor of Technology Honours in Engineering |
| Name of the Examination | : Final Examination |
| **Course Code and Title** | : **MHZ3531 / MPZ3231: Engineering Mathematics IA** |
| Academic Year | : 2017/18 |
| Date | : 30th January 2019 (Wednesday) |
| Time | : 0930 – 1230 hrs. |
| Duration | : **3 hours** |

**General Instructions**

1. Read all the instructions carefully before answering the questions.
2. This question paper consists of **Eight (8)** questions in **Four (4)** pages.
3. Answer any **Six (6)** questions only. All the questions carry equal marks.
4. Answer for each question should commence from a new page.
5. This is a Closed Book Test (CBT).
7. Answers should be in clear hand writing.
8. Do not use Red colour pen.

**Q1.** (a). State the Ceyley-Hamilton's theorem. (05%)

i). Let $A = \begin{pmatrix} 7 & 4 \\ -9 & -5 \end{pmatrix}$. Using the characteristic equation of $A$, prove that $A^2 - 2A + \bar{I} = \bar{0}$, where $\bar{I}$ and $\bar{0}$ are the unit matrix and the zero matrix of order two respectively. Hence, find a matrix $B$ such that $B^2 = \bar{0}$ and $B \neq \bar{0}$. (20%)

ii). Find the constants $\alpha$ and $\beta$ of the following equation. (15%)
$x^n = (x-1)^2 f(x) + \alpha x + \beta$, where $n$ is a positive integer such that $n > 1$ and $f(x)$ is a polynomial of degree $n-2$.

iii). Hence, deduce that for every $n \in \mathbb{Z}^+, A^n = \begin{bmatrix} 1+6n & 4n \\ -9n & 1-6n \end{bmatrix}$. (10%)

(b). Using the column operations and row operations, prove that (30%)

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -\frac{(a+b+c)}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2].$$

Hence, deduce that $\begin{vmatrix} a+b-c & b+c-a & c+a-b \\ b+c-a & c+a-b & a+b-c \\ c+a-b & a+b-c & b+c-a \end{vmatrix} = 4\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$. (20%)

**Q2.** (a). Let $y = f(x) = \dfrac{x^3}{3-x^2}$.

i). Find the vertical and slant asymptotes of the curve of $y = f(x)$. (20%)

ii). Prove that $\dfrac{dy}{dx} = -\dfrac{x^2(x-3)(x+3)}{\left(\sqrt{3}-x\right)^2\left(\sqrt{3}+x\right)^2}$. Determine the sign of $\dfrac{dy}{dx}$ as $x$ varies from $-\infty$ to $\infty$. Sketch the graph of $y = f(x)$. (50%)

(b). If $z = f(x,y)$, $x = rcos\theta$ and $y = rsin\theta$, then prove that (30%)

$$\left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r}\left(\frac{\partial z}{\partial \theta}\right)^2 = \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2.$$

**Q3.** (a). Using De Moivre's theorem, find $sin3\theta$ in terms of $sin\theta$ and $cos3\theta$ in terms of $cos\theta$. Deuce that $\dfrac{tan3\theta}{tan\theta} = \dfrac{tan^2\theta - 3}{3tan^2\theta - 1}$, where $\theta \neq n\pi, n \in \mathbb{Z}$. (30%)

i). Considering the general solutions of the equation $tan3\theta = 1$, prove that the roots of the equation $x^2 - 4x + 1 = 0$ are $tan\frac{\pi}{12}$ and $tan\frac{5\pi}{12}$. (20%)

ii). Show that $sec^2\dfrac{\pi}{12} + sec^2\dfrac{5\pi}{12} = 16$. (10%)

(b). $P(z)$ sitisfies the locus of $arg\left(\frac{z-4}{z+4}\right)=\frac{\pi}{4}$. Show that $Im(z)>0$ and Sketch the locus. Deduce that the above curve can be expressed as $|z-4i|=4\sqrt{2}$ and $Im(z)>0$. (40%)

**Q4.** (a). Prove that the equation of the normal to the parabola $y^2=4x$ at the point $P\ (t^2,2t)$ is $y+tx=2t+t^3$. (20%)

The normal to the parabola $y^2=4x$ at $P\ (t^2,2t)$ meets the axis of the parabola at $G$ and $GP$ is produced beyond $P$ to $Q$ so that $GP = PQ$.

i) Find the coordinates of the points $G$ and $Q$. (20%)

ii). Show that the locus of $Q$ is the parabola $y^2=16\ (x+2)$. (20%)

(b). Find the equations of two straight lines through the origin each of which intersects the straight line $\frac{x-3}{2}=\frac{y-3}{1}=\frac{z}{1}$ and inclined at an angle of $\frac{\pi}{3}$ to that straight line. (40%)

**Q5.** (a). Using an integrating factor, find the general solution of the differential equation $\frac{dy}{dx}+ytanx=secx.$ (30%)

(b). Using the substitution $y=vx$, find the general solution of the differential equation $2xy\frac{dy}{dx}=y^2-x^2$. (30%)

(c). Prove that the general solution of the differential equation (40%)

$\frac{d^2y}{dt^2}-2a\frac{dy}{dt}+(a^2+\omega^2)y=0$ is $y(t)=e^{at}(Acos\omega t+Bsin\omega t)$,

where $A$ and $B$ are arbitrary contants. If $y(0)=4$ and $y^1(0)=4a$, then deduce that the solution is $y(t)=4e^{at}cos(\omega t)$.

**Q6.** Define the vector product and scalar product of two non-zero vectors $\boldsymbol{a}$ and $\boldsymbol{b}$. (20%)

(a). The position vectors of the points $X$ and $Y$ are $\boldsymbol{x}$ and $\boldsymbol{y}$ respectively. If the line joining $X$ and $Y$ is divided internally into the ratio $m:n$ by the point $Z$, then prove that the position vectorof the point $Z$ is $\frac{1}{m+n}(n\boldsymbol{x}+m\boldsymbol{y})$. (20%)

(b). The point $O$ lies inside of the triangle $ABC$. The lines $AO$, $BO$ and $CO$ are extended and they meet the sides $BC$, $CA$ and $AB$ at the points $L$, $M$ and $N$ respectively. If BL : $LC = \lambda$: 1, $CM$: $MA = \mu$ : 1 and $AN : NB = \nu : 1$, then by using the vector product and part (a), show that $\lambda\mu\nu = 1$. (30%)

(c) Let $\boldsymbol{a} = 2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, $\boldsymbol{b} = 6\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ and $\boldsymbol{c} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$. (30%)

i). Find the unit vector perpendicular to both the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$.

ii). Find the angle between the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$.

iii). Find the triple scalar product of the vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$.

**Q8.** (a). State Bayer's theorem. A bag has 10 number of books that are identical, except only in colours. Also, nobody knows that how many white coloured books are in the bag. If one book was randomly drawn at one time from the bag, then it was a white. Find the probability that all the books in the bag are white coloured. (40%)

(b). The amount of rice (in hundred kilograms) that a supermarket sells in a day is a random variable with density function (60%)

$$f(x) = \begin{cases} k(3 - |x - 3|), & \text{if } 0 \le x \le 6 \\ 0, & \text{if } x < 0 \text{ or } x > 6 \end{cases}$$, where $k$ is a constant.

i). Find the value of $k$.

ii). What is the probability that the number of kilograms of rice that will be sold in a day is,

(α). More than 300 kilograms?

(β). Between 150 and 450 kilograms?

iii). The events in (α) and (β) are denoted by $A$ and $B$ respectively. Determine whether $A$ and $B$ are independent.

End

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**Q7.** (a). Using Newton-Raphson method, construct an iterative formula to calculate the square root of a positive number $c$. (60%)

Hence, taking $x_0 = 2$, calculate $\sqrt{5}$ correct to nine decimal places.

(b). Suppose that a function $f(x)$ acquires the following values for the given $x$ values in an experiment. (40%)

| $x$ | 0 | 0.1 | 0.2 | 0.3 | 0.4 |
|---|---|---|---|---|---|
| $f(x)$ | 1 | 1.01 | 1.04 | 1.09 | 1.16 |

Use the following rules to approximate the value of $\int_0^{0.4} f(x)dx$.

i). Trapezoidal rule.

ii). Simpson's rule.