The Open University of Sri Lanka
Faculty of Engineering Technology

Study Programme	:	Diploma in Technology/Bachelor of Technology (Engineering)
Name of the Examination	:	Final Examination
Course Code and Title	:	**MEX5233 Dynamics of Mechanical Systems**
Academic Year	:	2014/15
Date	:	09th August 2015
Time	:	0930hrs – 1230hrs
Duration	:	3 hours

General instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of 8 questions.
3. Answer any 5 questions selecting **at least TWO** from each section.

SECTION - A

QUESTION 01

A machine of mass m produces a disturbing force P sin ωt, where P is the magnitude and ω is the frequency. It is isolated from the ground by an isolator consisting of a spring of stiffness k and a damper having a damping ratio β. The motion of the machine is given, in usual notation, as

$$x = \frac{P}{m\omega_n^2}\frac{1}{\sqrt{\left(1-r^2\right)^2+4\beta^2 r^2}}\sin(\omega t-\phi)$$

(a) Show that the transmissibility ratio (T) of the force transmitted to the ground at steady state is given by

$$T=\left[\frac{1+(2\beta r)^2}{\left(1-r^2\right)^2+(2\beta r)^2}\right]^{1/2} \quad \text{where } r=\frac{\omega}{\omega_n} \quad \text{and} \quad \omega_n=\sqrt{\frac{k}{m}}.$$

(b) A machine of mass 1000 kg and operating at a speed of 1500 rev/min. produces a repeating force of magnitude 15 kN. The machine is isolated from a rigid foundation by an isolator. Calculate the maximum stiffness of an undamped isolator which will limit the force transmitted to the foundation to 2400 N.

(c) If the damping ratio of the isolator is 0.25, what would be the maximum stiffness of the isolator ?

QUESTION 02

Fig. Q2 shows a uniform rigid rod of length $3a$ and mass m pivoted at its left end. The rod is supported by a spring of stiffness $3k$ attached to the rod at a point at a distance $2a$ from the left end. A spring of stiffness k and carrying a mass m is attached to the right end of the rod, while another spring of stiffness k and carrying a mass m is attached to the rod at a distance a from the left end. The rod is horizontal in the equilibrium position.

(a) Determine the equations of motion for small oscillations of the system and show that the frequency equation can be expressed as

$$(\alpha - 1)(3\alpha^2 - 25\alpha + 12) = 0$$

where $\alpha = \dfrac{m\omega^2}{k}$ and ω is the natural frequency of oscillations.

(b) If $m = 1$ kg and $k = 2$ kN/m, evaluate the frequencies.

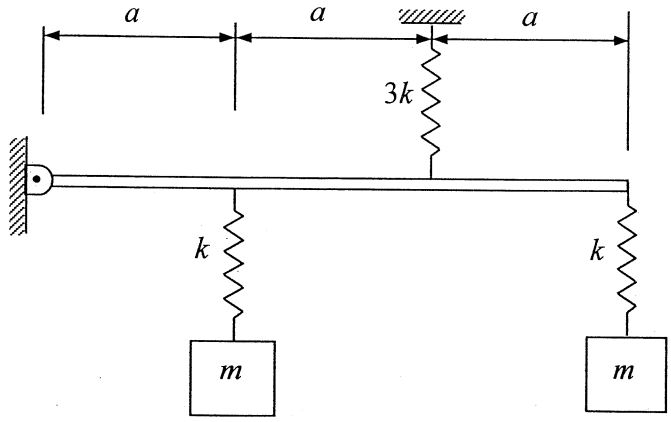


Fig. Q2

QUESTION 03

Fig. Q3 shows a torsional system in which three rotors of moments of inertia I_1, I_2 and I_3 are connected by shaft sections having torsional stiffnesses K_1, K_2, K_3 and K_4.

(a) Determine the equations of motion and obtain the frequency equation for small torsional oscillations.

(b) Show that, if $K_1 = K_2 = K$, $K_3 = K_4 = 3K$, $I_1 = I$, $I_2 = 2I$ and $I_3 = 3I$, the frequency equation can be expressed as $(\omega^2 - 2\lambda^2)(\omega^4 - 4\lambda^2\omega^2 + 2\lambda^4) = 0$ where $\lambda^2 = \frac{K}{I}$ and ω is the natural frequency of torsional oscillations.

(c) Determine the natural frequencies in terms of I and K and the mode shapes.

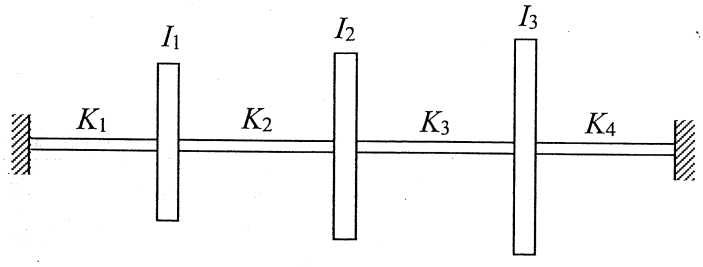


Fig. Q3

QUESTION 04

A uniform beam of length L, total mass m, and flexural rigidity EI is simply supported at its ends and carries a concentrated mass $3m$ at its mid span. The first mode of the deflection curve of the beam during free transverse vibrations is assumed to be a 'half a sine wave'.

(a) Using Energy method, show that the corresponding frequency of vibrations (ω) of the beam can be expressed as, $\omega = 3.726\sqrt{\frac{EI}{mL^3}}$.

(b) Hence determine the natural frequency of transverse vibration of the first mode for a uniform steel beam of diameter 20 mm and length 1.5 m, simply supported at its ends and carrying a concentrated mass, equal to three times of the mass of the beam, at mid span. The density and the modulus of elasticity of steel are 7.85×10^3 kg/m^3 and 200 GN/m^2 respectively.

SECTION - B

QUESTION 05

(a) Determine the damping ratio, undamped natural frequency and damped natural frequency for the speed control system in Fig. Q5(a). What is the response of this system to a unit step input when the initial conditions are zero ?

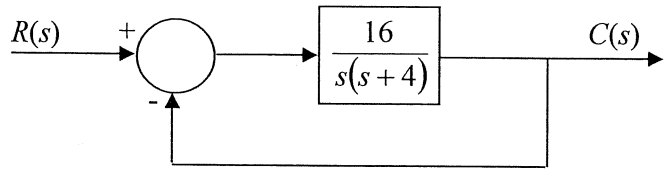


Fig. Q5(a)

(b) To improve the dynamic performance of the speed control system shown in Fig. Q5(a), derivative action is added to the feed back path as shown in Fig. Q5(b). Determine the value of K such that the resulting system will have a damping ratio of 0.8. What is the unit step response of this system if all the initial conditions are zero?

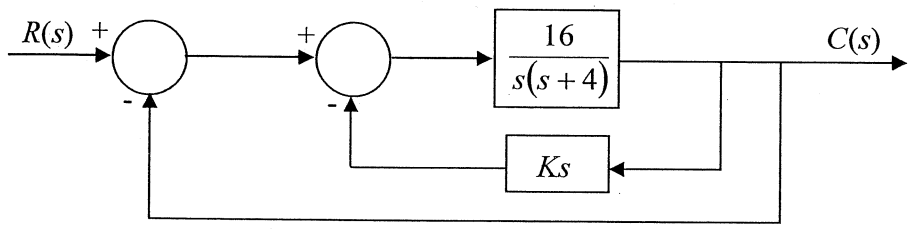


Fig. Q5(b)

(c) Compare the peak time and percentage overshoot in (a) and (b) above.

QUESTION 06

The open loop transfer function of a unity feedback control system is given by

$$G(s) = \frac{K}{s(s^2 + 6s + 13)}$$

where K is the gain of the controller.

(a) Draw the root locus as a function of K and determine the range of values of K for which the system is stable. Verify your result using Routh's criterion.

(b) Using the root locus you have drawn show that s = - 4 is a closed loop pole of the system when $K = 20$. Determine the other closed loop poles.

(c) Determine, using the root locus you have drawn, the damping ratio of the closed loop system when $K = 20$.

QUESTION 07

The open loop transfer function of a unity feed back control system is given by

$$G(s) = \frac{K}{s(1 + 0.1s)(1 + s)},$$

where K is the gain of the controller.

Draw the Bode plots for $\frac{G(j\omega)}{K}$ and determine the value of K in order to have a

(a) gain margin of 10 db

(b) phase margin of 30°.

QUESTION 08

Fig. Q8 shows a hydraulic servomechanism in which the "walking beam" connects the input position x, the valve position z and the piston position y. One end of the piston rod of the piston in the working cylinder is connected to a mass m which in turn is connected to a spring and a dashpot. The spring has a stiffness k and the damping constant of the dashpot is c. When z is zero, the valve is fully closed and the walking beam is vertical. The rate of flow of oil to and from the working cylinder, of which the piston has an area of cross section A, is q_1 and it is given by,

$q_1 = B_1 z$, where B_1 is a constant.

The leakage past the piston in the working cylinder is q_2 and it is given by

$q_2 = B_2 p$, where B_2 is a constant and p is the pressure difference across the piston..

Draw a complete block diagram of the overall system and obtain the transfer function relating x and y.

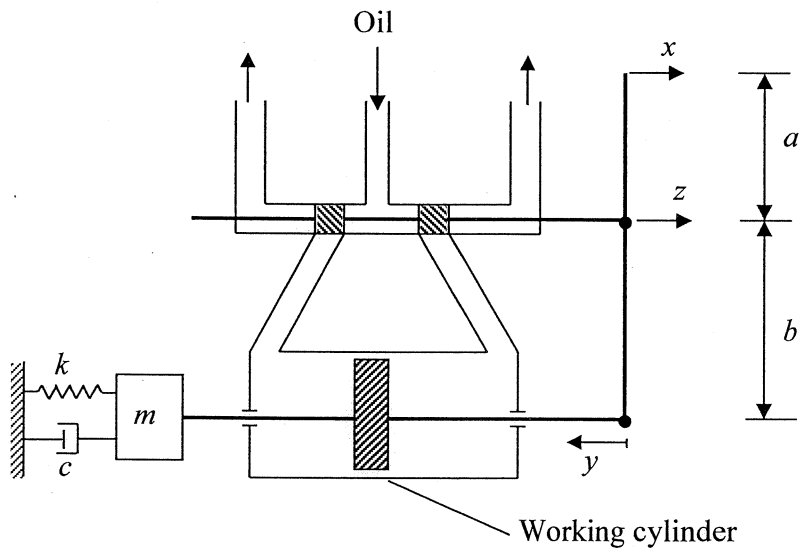


Fig. Q8

LAPLACE TRANSFORMS

TIME FUNCTION $f(t)$	LAPLACE TRANSFORM $F(s)$
Unit Impulse $\delta(t)$	1
Unit step	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
$\frac{df(t)}{dt}$	$sF(s)-f(0)$
$\frac{d^n f(t)}{dt^n}$	$s^n F(s) - s^{n-1} f(0) - s^{n-2} \frac{df(0)}{dt} \ldots - \frac{d^{n-1} f(0)}{dt^{n-1}}$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

END