(31) 00137

The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Mathematics and Philosophy of Engineering

Study Programme	: Bachelor of Technology Honours in Engineering
Name of the Examination	: Final Examination
Course Code and Title	**: MHJ5334 / MPJ5134: The Nature of Mathematics I**
Academic Year	: 2017/18
Date	: 22nd January 2019 (Tuesday)
Time	: 1330 – 1630 hrs.
Duration	: **3 hours**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **Seven (7)** questions in **Four (4)** pages.
3. Answer any **Five (5)** questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. This is a Closed Book Test (CBT).
7. Answers should be in clear hand writing.
8. Do not use Red colour pen.

Some important notes:

❖ ***Axioms for real numbers***

For arbitrary real numbers a, b, c :

A0. $a+b$ *and* $a.b$ *real numbers*

A1. $a+b=b+a$

A2. $a+(b+c)=(a+b)+c$

A3. $a+0=a$

A4. $a+(-a)=0$

M1. $a.b=b.a$

M2. $a.(b.c)=(a.b).c$

M3. $a.1=a$

M4. $a.\frac{1}{a}=1$ if $a \neq 0$

D. $a.(b+c)=a.b+a.c$

❖ ***Order Axioms***

P1 – If a,b *in* $\mathbb{P}$*, then* $a+b$ *in* $\mathbb{P}$

P2 – If a,b *in* $\mathbb{P}$*, then* $a.b$ *in* $\mathbb{P}$

P3 – For any two real numbers a,b *in* $\mathbb{R}$*, one and only one of the following is true:* a *in* $\mathbb{P}$*,* $a=0,-a$ *in* $\mathbb{P}$.

❖ *Integer* a *divide* $b\,(a|b)$ *if and only if* $b=c.a$ *for some integer* c.

❖ ***Mathematical Induction:***

If we can prove both of the following steps, then we know that the proposition is a result (Theorem), true for all n.

(I) The initial step:

Prove the proposition is true for $n=1$

(II) Inductive step:

Assume that the proposition is true for $n=k$*, and on this basis prove it is true for* $n=k+1$.

❖ ***Results of sets.***

Let A,B *and* C *be any three sub sets of the universal set* E.

Union $(\cup)$

U(1): $A\cup A=A$

U(2): $A\cup B=B\cup A$.

U(3): $A\cup\emptyset=A$

U(4): $A\cup(B\cup C)=(A\cup B)\cup C$

U(5): $A\cup B=B\Leftrightarrow A\subseteq B$.

Intersection $(\cap)$

Int(1): $A\cap A=A$

Int(2): $A\cap B=B\cap A$

Int(3): $A\cap\emptyset=\emptyset$

Int(4): $A\cap(B\cap C)=(A\cap B)\cap C$

Int(5): $A\cap B=A\Leftrightarrow A\subseteq B$.

Compliment

C(1): $(A^C)^C=A$

C(2): $(A\cup B)^C=A^C\cap B^C$

C(3): $(A\cap B)^C=A^C\cup B^C$

C(4): $A\cap A^C=\emptyset$

C(5): $A\cup A^C=E$

C(6): $A\subseteq B\Rightarrow B^C\subseteq A^C$

C(7): $A\backslash B=A\cap B^C$

C(8): $A\Delta B=(A\cap B^C)\cup(A^C\cap B)$

Distribution

Dist(1) $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$

Dist(1) $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

---------- **End of the notes** ----------

Q1 **(a)** Let a, b, x and y be real numbers. Prove that (40%)

i) $-(a+b) = (-a) + (-b)$.

ii) If $a + x = 0$ and $a + y = 0$, then prove that $x = y$. Using this result, deduce that $a = -(-a)$.

(b) Let $a \in \mathbb{Z}$. If $3|a^2$, then prove that $3|a$. (30%)

(c) Prove that there is no rational number whose square is equal to 3. (30%)

Q2 **(a)** Let x, y and z be real numbers. Prove that (30%)

i) If $x \geq y$, then $x + z \geq y + z$.

ii) If $x \geq y$ and $z > 0$ then $xz \geq yz$.

(b) Find the set of all solutions of the inequality $|x - 4| < 8 - |x + 2|$, and draw the related graph, where x is a real number. (50%)

(c) Let a be a real number. Prove that $|a| \geq 0$ and $|a| = 0 \Leftrightarrow a = 0$. (20%)

Q3 **(a)** Let p and q be two propositions. By using truth tables, prove the following. (20%)

i) $not(p \Rightarrow q) \equiv p \wedge notq$. ii) $not(p \wedge q) \equiv p \Rightarrow notq$.

(b) Consider the following statement. φ : "For all x and for all y in the real number system, $(x + y)^2$ is greater than $(x - y)^2$ if and only if xy is greater than zero". (40%)

i) Write down φ using quantifiers and symbols. Deduce the symbolic form of $not\varphi$.

ii) Prove that φ is true.

(c) Consider the following statement. ψ : "There exist x and y in the real number system such that $(x + y)^2$ is less than $(x - y)^2$ and xy is greater than or equal to zero". (40%)

i) Write down ψ using quantifiers and symbols. Deduce the symbolic form of $not\psi$.

ii) Prove that ψ is false.

Q4 (a) Let a, b and c be integers. Show that if $a|b$ and $a|$c, then $a|5b + 7c$. (30%)

(b) Show that if $3 \nmid a$, then $3|a^2+2$. (30%)

(c) Let n be a positive integer. Using the Mathematical Induction, prove that (40%)

$$1^2+2^2+3^2+\cdots+(n-1)^2+n^2=\frac{1}{6}n(n+1)(2n+1) \text{ for all } n.$$

Q5 Let A, B and C be three sub sets of the universal set E.

(a) Give examples to show that the following statements are false. (20%)

i) $A \cup B = A \cup C \Rightarrow B = C$. ii) $A \cap B = A \cap C \Rightarrow B = C$.

(b) Using the given results of sets in the first page, prove that (40%)

i) $A \cap E = A$. ii) $A \cup B = B \Leftrightarrow A \cap B^C = \emptyset$.

(c) Using the Venn-diagram method, prove that $(A \cup B)^C = A^C \cap B^c$. (20%)

(d) Using the logic method, prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. (20%)

Q6 (a) Define the product $A \times B$ of the sets A and B. Prove that (50%)

i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

ii) $A \times (B \backslash C) = (A \times B) \backslash (A \times C)$.

(b) Let $A = \{1, 2, 3, 4\}$. Define a relation ρ on A and for any $x, y \in A$ (50%)

$x \rho y \Leftrightarrow x + y$ is even

i) Write down $A \times A$ and the set R, where $R = \{(x, y) | \, x \rho y, x, y \in A\}$.
State whether $A \times A = R$ or not. Justify your answer.

ii) Express ρ^{-1} as a set of ordered pairs.

iii) Show that ρ is reflexive, symmetric, and transitive.

Q7 (a) i) State the division Algorithm for integers a, b with $b > 0$. (50%)

ii) Appling the division Algorithm, find the $g.c.d(1356, 414)$. Find the integers u and v such that $gcd(1356, 414) = 1356u + 414v$.

(b) i) State the well-ordering principle. (50%)

ii) Using above principle of $\mathbb{N}$, show that $\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$.

End

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