THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Mathematics & Philosophy of Engineering

Bachelor of Technology Honors in Engineering /
Bachelor of Software Engineering Honors

Final Examination (2017/2018)
MHZ5340 / MHZ5360 / MPZ5140 / MPZ5160: Discrete Mathematics II

Date: 16th January 2019 (Wednesday) **Time: 13:30 pm – 16:30 pm**

Instruction:

- **Answer only six questions.**
- **Please answer a total of six questions choosing at least one from each single section.**
- **All symbols are in standard notation and state any assumption that you made.**

SECTION – A

Q1.

I. Define a semi-group in usual notation.

Let " $*$ " be the binary operation on $\mathbb{Z}$ defined by the following ways:

a) "$a * b = a^b$ for all $a, b \in \mathbb{Z}$;
b) "$x * y = max(x, y)$" for all $x, y \in \mathbb{Z}$;

Verify that where $(\mathbb{Z}, *)$ a semi-group is for each of the above case. **[30%]**

II. Determine whether $(\mathbb{N}, -)$ is a semi-group or not. **[10%]**

III. A binary operation "$*$" on $\mathbb{Q}$ is defined by $m * n = m + n - 2mn$. **[35%]**

a) Verify that $(\mathbb{Q}, *)$ a semi-group. Is it commutative?
b) Determine the identity element of $(\mathbb{Q}, *)$.

IV. Let the operation "#" is defined on the set of real number $\mathbb{R}$ as follows:

$$a\#b = ab + 3 \text{ for all } a, b \in \mathbb{R}.$$

Prove that "#" is the commutative binary operation on $\mathbb{R}$, but $(\mathbb{R}, \#)$ is not a semi group. **[25%]**

Q2.

I. Define a group and an Abelian group with usual notation **[20%]**

II. Let $G = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \mid ac \neq 0,\ a, b, c \in \mathbb{R} \right\}$ and " $*$ " denotes the usual matrix multiplication.

a) Show that $(G,\ *)$ is a group.

b) Determine whether $(G,\ *)$ is an Abelian or not. **[60%]**

III. Let $(B, @)$ be a group, where $x@y = \frac{3(x+y)}{13}$, for all $x, y \in \mathbb{R}$. Show that $(B, @)$ is an Abelian group. **[20%]**

Q3.

I. Let $A = \{-1,1\}$. Show that $(A,*)$ is a group where " $*$ " is the ordinary multiplication. **[30%]**

II. Define a Homomorphism and Isomorphism for group in usual notation. **[10%]**

In the following, verify that each of the mappings $\emptyset_i$, defined from G to $\bar{G}$ is a homomorphism for $i = 1, 2, 3, 4$. **[60%]**

a) $G = \bar{G}$ is the group of non-zero real numbers under multiplication, $\phi_1 = x^2$ for all $x \in G$.

b) $G = \bar{G}$ is the group of non-zero real numbers under multiplication, $\phi_2 = 2^x$ for all $x \in G$.

c) G is the group of real numbers under addition, $G = \bar{G}$, $\phi_3 = x + 1$ for all $x \in G$.

d) G is the group of real numbers under addition, $G = \bar{G}$, $\phi_4 = 13x$ for all $x \in G$.

SECTION – B

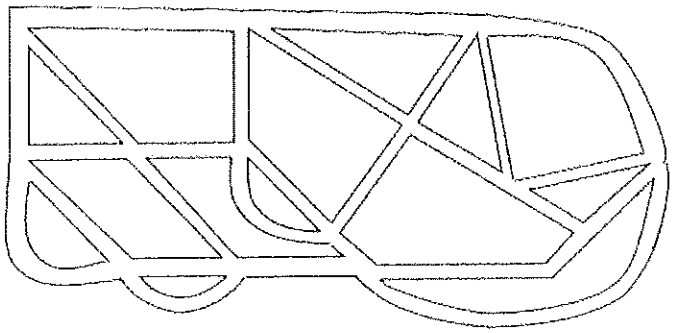
Q4.

I. By drawing each of the following graph, determine whether which of these graphs are simple or not

a) $G_1 = \{V_1, E_1\}$, where $V_1 = \{1, 2, 3, \ldots, 7, 8\}$ and $E_1 = \{\{x, y\} \mid x \times y \text{ is divided by } 2, x < y\}$. **[20%]**

b) $G_2 = \{V_2, E_2\}$, where $V_2 = \{1, 2, 3, \ldots, 9, 10\}$, such that two number "$i$" and "$j$" are adjacent if and only if $i + j$ is a multiple of 4 and $i < j$. **[20%]**

II. The diagram below is a map of the road system in a town. Draw a multigraph to model the road system, using a vertex to represent a junction and edge represent a road joining two junctions. **[20%]**



III. Find the number of vertices of a complete graph which has at least 2000 edges. **[20%]**

IV. What is the largest possible number of vertices in a graph with 2019 edges if all vertices have degree at least three? **[20%]**

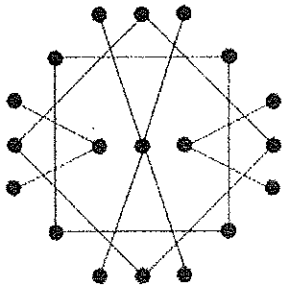
Q5.

I. Let G be a graph of 10 vertices and 21 edges in which every vertex is of degree 3 or 5. How many vertices of degree 3 and 5 does G have? Construct one such graph G. **[30%]**

II. Let G be a graph of 8 vertices and 13 edges. Denote by n_i the number of vertices in G of degree i, where $i = 2, 3, 4$. Assume that $n_3 \geq 1$. Find all possible answers for (n_2, n_3, n_4). For each your answer, construct a corresponding graph. **[50%]**

III.

a) Is the following graph disconnected? If it is so, find its number of components. **[10%]**



b) Let G be a graph with $V(G) = \{1, 2, \ldots, n\}$, where $n \geq 5$, such that two numbers i and j in $V(G)$ are adjacent if and only if $|i - j| = 5$. How many components does G have? **[10%]**

Q6.

I. G is the graph whose adjacency matrix A is given by

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

a) Let $V(G) = \{a_1, a_2, a_3, a_4\}$. Find the number of paths of length **three** joining vertices a_3 and a_2. Give the list of paths if any. **[40%]**

b) Without drawing a diagram of G, determine whether G is connected or not. **[20%]**

c) Draw the graph corresponding to the of adjacency matrix A. **[10%]**

II. Define a "Tree" graph.

a) Draw a tree with 13 vertices, exactly 09 of which have degree of one.

b) Draw a tree 20 vertices at least haft of which have degree 1. **[30%]**

SECTION – C

Q7.

I. Iterate the Eco-system grow model for the relation $y_{n+1} = \lambda y_n$ with $y_0 = 0.3$ and $\lambda = 0.8$ (at least 8 iteration steps are necessary) and draw the diagram for the relation. **[20%]**

II. For the given relationship $t_{n+1} = \lambda t_n(1 - t_n)$, where $\lambda = 1.4$, obtain the convergent value for each of the following case: **[25%]**

a) $t_0 = 0.3$,

b) $t_0 = 0.8$.

III. Draw the graph for the relation $Z_{n+1} = Z_n^2$. Find Z_5 and hence deduce Z_n as $n \to \infty$ for each of following case: **[55%]**

a) $Z_0 = 1.5 + 0.3i$,

b) $Z_0 = 0.4 + 0.3i$,

c) $Z_0 = i$.

Q8.

A three dimensional system is governed by the following system of differential equations:

$$\frac{dx}{dt} = x + 2y + z\,,$$

$$\frac{dy}{dt} = -2x + y + z,$$

$$\frac{dz}{dt} = 4x + 2y\,,$$

where , y , and z are function of t and at $t = 0, (x,\ y, z) = (0, 1, 1)$.

Find the phase space value $(x(t),\ y(t), z(t)$)for $t = 1, 2$. **[100%]**

Q9.

I. Let $L_1 = \{a, ab, b^2\}$ and $L_2 = \{b^2, aba\}$ be languages. Find **[15%]**
a) L_1L_2, b) L_2L_1, c) L_1^2.

II. Show that the string $babaaaba$ is a sentence governed by the grammar G, where $G = \{\{S, A, B\}, \{a, b\},\ P, S\}$ and starting symbol S and production P, **[15%]**

$$P = \{S \to aABa, A \to baABb, B \to Aab, aA \to baa, bBb \to abab\}.$$

III. Construct the production rules to generate each of the following language: **[20%]**

a) $L(G_1) = \{a^n b^m c^l : n, m, l \geq 1\}$,

b) $L(G_2) = \{(ab)^n : n \geq 1\}$

IV. Let M be a finite state machine with state table given below: **[20%]**

State	Input			Output		
	a	***b***	***c***	***a***	***b***	***c***
S_0	S_0	S_1	S_2	0	1	0
S_1	S_1	S_1	S_0	1	1	1
S_2	S_2	S_1	S_0	1	0	0

Draw the state diagram and find the output string for the input string ***aabbcc***. Initial state S_0.

V. Let M be a Mealy machine. Let $s \in S, a, b \in I$ and $s \in I^*$ and defined functions

$\delta: S \times I^* \to S$ and $\beta^*: S \times I^* \to O^*$ by

$\delta^*(s, \Omega) = s,$

$\delta^*(s, a.x') = \delta^*[\delta(s, a), x'],$

$\beta^*(s, \Omega) = \Omega,$

$\beta^*(s, a.x') = \beta(s, a)\beta^*[\delta(s, a), x'].$

The two frame binary pipeline devise hold up two binary as in the following table,

State	**Input**		**Output**	
	a	*b*	*a*	*b*
00	01	11	1	1
01	01	10	0	1
10	00	00	0	1
11	01	10	1	0

Find the two frame binary pipeline buffer and work out its response to the sequence *baababa* from the state 01. **[30%]**