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The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Mathematics and Philosophy of Engineering

Study Programme	: Bachelor of Technology Honours in Engineering
Name of the Examination	: Final Examination
Course Code and Title	**: MHZ5530/MPZ5230 Engineering Mathematics III**
Academic Year	: 2017/18
Date	: 24th January 2019
Time	: 0930 - 1230hrs
Duration	: **3 hours**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **Nine (9)** questions in **Five (5)** pages.
3. Answer any **Six (6)** questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Relevant statistical tables are provided.
6. This is Closed Book Test (CBT).
7. Answers should be in clear hand writing.
8. Do not use Red colour pen.

Question 01

a) Find the value of the volume integral of the function $f(x, y, z) = y + z$, over the region inside the circular cylinder $x^2 + y^2 = 1$ and lying between the planes $z = 0$ and $z = 1$.

[30 %]

b) Determine the volume of the region that lies behind the plane $x = 4 - y - z$ and in front of the region in the yz plane that is bounded by $z = \frac{1}{2}\sqrt{y}$ *and* $z = \frac{1}{4}y$. [30 %]

c) Evaluate $\iiint_B \sqrt{x^2 + y^2 + z^2}\, dV$, where B is the region bounded by the plane $z = 3$ and the cone $z = \sqrt{x^2 + y^2}$. [40 %]

Question 02

a) The velocity of a fluid is given by the vector field $\underline{u}(r, \theta, z) = \frac{1}{r}\underline{e}_r, \; for \; (r \neq 0)$. Find the flux of the fluid ($\int_S \underline{u} \cdot \underline{n}\, dA$), through the curved surface S of the cylinder $r = R$ between $z = 0$ and $z = 1$, where $\underline{n}$ is the unit outward normal to the surface S. [30 %]

b) i. State the Divergence Theorem. [15 %]

ii. Verify the Divergence Theorem for the vector filed

$\underline{F} = 3x^2yz\underline{i} + 3xy^2z\underline{j} + 6xyz^2\underline{k}$

on the region G given by, $0 \leq x \leq a, 0 \leq y \leq b$ and $0 \leq z \leq c$ [55 %]

Question 03

a) State Stokes' Theorem. [20 %]

b) Evaluate $\oint_C \underline{F}.d\underline{r}$, where $\underline{F} = z^2\underline{i} + y^2\underline{j} + x\underline{k}$, and c is a triangle with vertices (1,0,0), (0,1,0) and (0,0,1) in counter-clockwise rotation. [80 %]

Question 04

a) Define an irrotational vector field. [10 %]

b) Write down any two properties of an irrotational vector field. [20 %]

c) Show that $\underline{F} = (6x^2yz + 3y^2)\underline{i} + (2x^3z + 6xy + 3z)\underline{j} + (2x^3y + 3y)\underline{k}$ with domain $\mathbb{R}^3$ is an irrotational vector. [30 %]

d) Find a scalar potential function ϕ such that $\underline{F} = grad\phi$. Here consider $\underline{F}$ defined in part (c). [30 %]

e) Find the line integral of $\underline{F}$ around the closed path $c: x^2 + y^2 = 4$. Here consider $\underline{F}$ defined in part (c). [10 %]

Question 05

a) Find the image of the rectangular region in z plane bounded by straight lines $x = 1$, $x = 2, y = 1$, and $y = 1$ under the mapping $w = \frac{1}{z}$. [20 %]

b) i. Find the Taylor series expansion of the function, $f(z) = Ln(1 + z)$ about $z_0 = 0$. [30 %]

ii. Determine the region of convergent for the series in part (b).(i) . [20 %]

iii. Expand $Ln\left(\frac{1+z}{1-z}\right)$ in the Maclaurin series. [20 %]

c) Find the Laurent series of $f(z) = \frac{z}{(z+1)(z+2)}$ about $z = -2$, valid for $0 < |z + 2| < 1$. [10 %]

Question 06

The moment of inertia tenser is given by,

$$T_{ij} = \begin{bmatrix} 6 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 6 \end{bmatrix}.$$

a) Find the eigenvalues and the principle axes of the tensor. [60 %]

b) Express the tensor with respect to its principle axes. [10 %]

c) Write the equation of the transformation relating to the coordinates x_i determined by the principle axes. [10 %]

d) Verify the results in (b), by using the transformation law for second order tensor. [20 %]

Question 07

a) Solve the following initial and boundary value problem using the variable separable method.

$$\frac{\partial^2 y}{\partial t^2} = 4\,\frac{\partial^2 y}{\partial x^2}, \text{ where } 0 \le x \le \pi \text{ and } t > 0$$

The boundary conditions are

$$y(0, t) = 0 \quad , \qquad y(\pi, t) = 0$$

The initial conditions are

$$\frac{\partial y}{\partial t}(x, 0) = 0 \text{ and } y(x, 0) = x(\pi - x), \text{ where } 0 \le x \le \pi \text{ and } t > 0 \qquad [50\ \%]$$

b) Apply Fourier transform to solve:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \qquad for\ x \geq 0, t \geq 0,$$

with the initial and boundary conditions,

i. $\frac{\partial u}{\partial x}(0,t) = -2 \qquad for\ t > 0$

ii. $u(x,0) = \begin{cases} 1 & 0 < x < 1 \\ 0 & x \geq 1 \end{cases}$

$u \to 0$ uniformly in t, as $x \to \pm\infty$.

[50 %]

Question 08

a) A person has 2000 acres of land on which he can grow corn, wheat and rice. Each acre of corn costs Rs.100 for preparation, 7 man-days of work, and yields a profit of Rs. 30. An acre of wheat costs Rs.120; requires 10 man-days work, and yield a profit of Rs.40. An acre of rice costs Rs.120 to prepare, requires 8 man-days work, and yields a profit of Rs.20. If the person has Rs.1 lakh for preparation and can count on 8000 man-days work. How many acres should be allocated to each crop to maximize the profit?

i. Identify the decision variables of the above problem. [10 %]

ii. Formulate the linear programming model. [20 %]

b) Consider the following linear programming problem.

$$\text{Maximize } z = 20x_1 + 30x_2$$

Subject to

$$2x_1 + 3x_2 \leq 120$$

$$x_1 + x_2 \leq 35$$

$$2x_1 + 1.5x_2 \leq 90$$

$$x_1, x_2 \geq 0$$

i. Solve above LP problem by Simplex Method. [45 %]

ii. Verify the solution by graphical method. [25 %]

Question 09

A lecturer wishes to test three different teaching methods I, II and III. To do this, three groups of 6 students each are chosen at random, and each group is taught by a different method. The same examination is then given to all the students, and the marks are shown in the following table. Determine at the

a) 5% significance level [80 %]

b) 1% significance level whether there is a significant difference in the teaching methods. [20 %]

Method I	85	62	71	58	73	65
Method II	81	85	68	92	90	80
Method III	73	79	60	75	81	62

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TABLE A.3

F Distribution: Critical Values of *F* (5% significance level)

ν_1 / ν_2	1	2	3	4	5	6	7	8	9	10	12	14	16	18	20
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	243.91	245.36	246.46	247.32	248.01
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.42	19.43	19.44	19.45
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.71	8.69	8.67	8.66
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.87	5.84	5.82	5.80
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.64	4.60	4.58	4.56
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.96	3.92	3.90	3.87
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.53	3.49	3.47	3.44
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.24	3.20	3.17	3.15
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.03	2.99	2.96	2.94
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.86	2.83	2.80	2.77
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.74	2.70	2.67	2.65
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.64	2.60	2.57	2.54
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.55	2.51	2.48	2.46
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.48	2.44	2.41	2.39
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.42	2.38	2.35	2.33
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.37	2.33	2.30	2.28
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.33	2.29	2.26	2.23
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.29	2.25	2.22	2.19
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.26	2.21	2.18	2.16
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.22	2.18	2.15	2.12
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.20	2.16	2.12	2.10
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.17	2.13	2.10	2.07
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.15	2.11	2.08	2.05
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.13	2.09	2.05	2.03
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.11	2.07	2.04	2.01
26	4.22	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.09	2.05	2.02	1.99
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.08	2.04	2.00	1.97
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.06	2.02	1.99	1.96
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.05	2.01	1.97	1.94
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.04	1.99	1.96	1.93
35	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16	2.11	2.04	1.99	1.94	1.91	1.88
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.95	1.90	1.87	1.84
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.95	1.89	1.85	1.81	1.78
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.86	1.82	1.78	1.75
70	3.98	3.13	2.74	2.50	2.35	2.23	2.14	2.07	2.02	1.97	1.89	1.84	1.79	1.75	1.72
80	3.96	3.11	2.72	2.49	2.33	2.21	2.13	2.06	2.00	1.95	1.88	1.82	1.77	1.73	1.70
90	3.95	3.10	2.71	2.47	2.32	2.20	2.11	2.04	1.99	1.94	1.86	1.80	1.76	1.72	1.69
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.85	1.79	1.75	1.71	1.68
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.78	1.73	1.69	1.66
150	3.90	3.06	2.66	2.43	2.27	2.16	2.07	2.00	1.94	1.89	1.82	1.76	1.71	1.67	1.64
200	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.80	1.74	1.69	1.66	1.62
250	3.88	3.03	2.64	2.41	2.25	2.13	2.05	1.98	1.92	1.87	1.79	1.73	1.68	1.65	1.61
300	3.87	3.03	2.63	2.40	2.24	2.13	2.04	1.97	1.91	1.86	1.78	1.72	1.68	1.64	1.61
400	3.86	3.02	2.63	2.39	2.24	2.12	2.03	1.96	1.90	1.85	1.78	1.72	1.67	1.63	1.60
500	3.86	3.01	2.62	2.39	2.23	2.12	2.03	1.96	1.90	1.85	1.77	1.71	1.66	1.62	1.59
600	3.86	3.01	2.62	2.39	2.23	2.11	2.02	1.95	1.90	1.85	1.77	1.71	1.66	1.62	1.59
750	3.85	3.01	2.62	2.38	2.23	2.11	2.02	1.95	1.89	1.84	1.77	1.70	1.66	1.62	1.58
1000	3.85	3.00	2.61	2.38	2.22	2.11	2.02	1.95	1.89	1.84	1.76	1.70	1.65	1.61	1.58

TABLE A.3 (continued)

F Distribution: Critical Values of *F* (1% significance level)

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	14	16	18	20
1	4052.18	4999.50	5403.35	5624.58	5763.65	5858.99	5928.36	5981.07	6022.47	6055.85	6106.32	6142.67	6170.10	6191.53	6208.73
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	99.42	99.43	99.44	99.44	99.45
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.05	26.92	26.83	26.75	26.69
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.37	14.25	14.15	14.08	14.02
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.89	9.77	9.68	9.61	9.55
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.60	7.52	7.45	7.40
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.36	6.28	6.21	6.16
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.56	5.48	5.41	5.36
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	5.01	4.92	4.86	4.81
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.60	4.52	4.46	4.41
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.29	4.21	4.15	4.10
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.05	3.97	3.91	3.86
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.86	3.78	3.72	3.66
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.80	3.70	3.62	3.56	3.51
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.56	3.49	3.42	3.37
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.45	3.37	3.31	3.26
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.35	3.27	3.21	3.16
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.27	3.19	3.13	3.08
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.19	3.12	3.05	3.00
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.13	3.05	2.99	2.94
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.07	2.99	2.93	2.88
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	3.02	2.94	2.88	2.83
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.97	2.89	2.83	2.78
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.93	2.85	2.79	2.74
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.99	2.89	2.81	2.75	2.70
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.96	2.86	2.78	2.72	2.66
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.93	2.82	2.75	2.68	2.63
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.90	2.79	2.72	2.65	2.60
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.87	2.77	2.69	2.63	2.57
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.74	2.66	2.60	2.55
35	7.42	5.27	4.40	3.91	3.59	3.37	3.20	3.07	2.96	2.88	2.74	2.64	2.56	2.50	2.44
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.56	2.48	2.42	2.37
50	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78	2.70	2.56	2.46	2.38	2.32	2.27
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.39	2.31	2.25	2.20
70	7.01	4.92	4.07	3.60	3.29	3.07	2.91	2.78	2.67	2.59	2.45	2.35	2.27	2.20	2.15
80	6.96	4.88	4.04	3.56	3.26	3.04	2.87	2.74	2.64	2.55	2.42	2.31	2.23	2.17	2.12
90	6.93	4.85	4.01	3.53	3.23	3.01	2.84	2.72	2.61	2.52	2.39	2.29	2.21	2.14	2.09
100	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59	2.50	2.37	2.27	2.19	2.12	2.07
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.23	2.15	2.09	2.03
150	6.81	4.75	3.91	3.45	3.14	2.92	2.76	2.63	2.53	2.44	2.31	2.20	2.12	2.06	2.00
200	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.60	2.50	2.41	2.27	2.17	2.09	2.03	1.97
250	6.74	4.69	3.86	3.40	3.09	2.87	2.71	2.58	2.48	2.39	2.26	2.15	2.07	2.01	1.95
300	6.72	4.68	3.85	3.38	3.08	2.86	2.70	2.57	2.47	2.38	2.24	2.14	2.06	1.99	1.94
400	6.70	4.66	3.83	3.37	3.06	2.85	2.68	2.56	2.45	2.37	2.23	2.13	2.05	1.98	1.92
500	6.69	4.65	3.82	3.36	3.05	2.84	2.68	2.55	2.44	2.36	2.22	2.12	2.04	1.97	1.92
600	6.68	4.64	3.81	3.35	3.05	2.83	2.67	2.54	2.44	2.35	2.21	2.11	2.03	1.96	1.91
750	6.67	4.63	3.81	3.34	3.04	2.83	2.66	2.53	2.43	2.34	2.21	2.11	2.02	1.96	1.90
1000	6.66	4.63	3.80	3.34	3.04	2.82	2.66	2.53	2.43	2.34	2.20	2.10	2.02	1.95	1.90