THE OPEN UNIVERSITY OF SRI LANKA

Bachelor of Technology (Level 05)

Final Examination (2014/2015)

MPJ 5134-The Nature of Mathematics -I

Duration: Three Hours

Date:9-8-2015 Time: 13.30pm -16.30pm

Instructions

- Answer Only five Questions
- This paper contains 5 pages
- State clearly any assumptions you required
- All symbols are in standard notations unless they are defined

1. Let a,b,c and d be real numbers.

   (a) Prove the following using field or order axioms:

   i. $-(a+b) = (-a) + (-b)$,

   ii. If $a \neq 0$ and $ab = ad$ then $b = d$,

   iii. If $b \neq 0$, $d \neq 0$ then $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$,

   iv. If $0 < a < b$ then $\frac{1}{b} < \frac{1}{a}$. [60%]

   (b) Let n be a positive integer. Prove that n is even if and only if $7n + 4$ is even. [25%]

   (c) Prove that $m^2 = n^2$ if and only if $m = n$ or $m = -n$. [15%]

2. (a) Let a and b be real numbers. Prove the following:

   i. $-|a| \leq a \leq |a|$,

   ii. If $b > 0$ then $|a| \leq b$ if and only if $-b \leq a \leq b$,

   iii. $|a+b| \leq |a| + |b|$. [60%]

   (b) Solve the following absolute value inequalities:

   i. $|x - \frac{1}{9}| \geq \frac{2}{9}$.

   ii. $\frac{2}{25} < \frac{3}{2+|7x-5|}$. [40%]

3. (a) Let a,b and c be any integers. Prove the following:

   i. If $a|b$ and $b \neq 0$, then $|a| \leq |b|$,

   ii. If $a|b$ and $b|c$ then $a|c$,

   iii. If $a|b$ and $a|c$ then $a|(b+c)$ and $a|(b-c)$. [60%]

   (b) Using principle of mathematical induction, prove the following:

   i. $3^{4n+2} + 5^{2n+1}$ is a multiple of 14, for all integers, $n \geq 0$.

   ii. $2n + 1 < 2^n$ for $n \geq 3$. [40%]

4. (a) Let p, q and r be propositions. With usual notations, Prove the following:

   i. $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

   ii. $\sim (p \vee q) \equiv \sim p \wedge \sim q$

   iii. using only properties of logical equivalence,
   $(p \wedge q) \wedge [(q \wedge \sim r) \vee (p \wedge r)] \equiv \sim (p \rightarrow \sim q)$. [45%]

   (b) Find the truth values of the following statements:

   i. $(e < \pi) \wedge (57$ is prime),

   ii. $\sqrt{2}$ is rational $\vee \left(\frac{1+\sqrt{(5)}}{2} < 2\right)$,

   iii. $(5^2 < 0) \rightarrow (1 < 2)$. [30%]

   (c) Using contrapositive law, prove that if the integer n^2 is a multiple of 3, then n is a multiple of 3. [25%]

5. (a) Define what is meant by an argument $P_1, P_2, \ldots P_n \vdash Q$ is valid?. [10%]

   (b) Let p,q, and r be propositions. Test the validity of the following arguments:

   i. $p \to q$,$(p \wedge q) \to r \vdash (q \to r)$. [20%]

   ii. If prices are high then wages are high. Prices are high or there price controls. If there are price controls there is no inflation. There is inflation. Therefore wages are high. [30%]

   (c) Let $Q(x, y) : x + y = x - y$ where the universe of discourse for x and y consist of all integers. Find the truth values of each of the following statements:

   i. $Q(1, 1)$,

   ii. $Q(2, 0)$,

   iii. $\forall y Q(1, y)$,

   iv. $\exists Q(x, 2)$. [40%]

6. (a) Let A,B and C be three sets. Prove the following by using Venn diagram method.

   i. $(A \cap B)^c = A^c \cup B^c$,

   ii. If $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$. [40%]

   (b) Let A, B, and C be subsets of U.

   i. Prove that $(A \setminus B) \setminus C = A \setminus (B \setminus C)$,

   ii. Given that $A \cap B$=$A \cap C$ and $A^c \cap B = A^c \cap C$. Prove or disprove that $B = C$.

   iii. Prove that $(A \triangle B)^c = A \triangle (B^c)$. [60%]

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7. (a) **Define the term "equivalence Relation" on a set.** [10%]

   i. **Let R be a binary relation defined on $R = \{(a, b) \in \mathbb{R} : a - b \leq 3\}$. Determine whether R is reflexive, symmetric, anti-symmetric or transitive.** [30%]

   ii. **Suppose that ρ is a relation on set of natural numbers and it is defined by $x\rho y$ if and only if $x - y$ is divisible by 5. Prove that ρ is an equivalence relation on $\mathbb{N}$.** [20%]

   (b) **Let $\gcd(a, b) = 1$. Prove the following:**

   i. $\gcd(2a + b, a + 2b) = 1$ **or** 3,

   ii. $\gcd(a + b, a^2 + b^2) = 1$ **or** 2. [40%]