00229

**THE OPEN UNIVERSITY OF SRI LANKA**

**BACHELOR OF TECHNOLOGY – LEVEL 05**

**FINAL EXAMINATION – 2014/2015**

**MPZ5230 – ENGINEERING MATHEMATICS III**

**DURATION: THREE (03) HOURS**

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**Date: 25th August 2015** **Time: 1330hrs – 1630hrs**

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**Instructions:**

- Answer only six (6) questions.
- State any assumptions you required.
- This paper contains five (5) pages.
- All symbols are in standard notation.

1. (a) Let $\vec{H} = (y^2 - 2xyz^3)\boldsymbol{i} + (3 + 2xy - x^2z^3)\boldsymbol{j} + (6z^3 - 3x^2yz^2)\boldsymbol{k}$.

   i. Evaluate $Curl\,\vec{H}$ and $div\,\vec{H}$,

   ii. Find $\varphi$ such that $\vec{H} = -grad\varphi$ where $\varphi(1,-1,2) = 2$.

   What can you say about the vector function $\vec{H}$ ? [50%]

   (b) Parabolic Cylindrical Coordinate $(u, v, z)$ is defined by the equations

   $$x = \frac{1}{2}(u^2 - v^2),$$

   $y = uv$, $z = z$. Express $\nabla^2\Phi = 0$ for this coordinate system, and find all its solutions of the form $\Phi = f(u^2 + v^2)$. [50%]

2. (a) Using plane polar coordinates, find the area in the XY-plane, outside the circle

$x^2 + y^2 = a^2$ and inside the circle $x^2 + y^2 - 2ax = 0$. [30%]

(b) State the Green's theorem in a plane. [20%]

(c) Show, by direct integration, that $\oint_C \sqrt{x^2 + y^2}\, dy = \frac{a^2}{2}(\sqrt{2} - 1)$, where $C$ is the closed curve formed by the lines $y = x$, $y = a$ and $x = 0$, described in the anti-clockwise sense. Hence verify the Green's theorem, by evaluating the corresponding surface integral over the area in the XY-plane bounded by $C$.

[50%]

3. (a) Show that

$$\iint_R r^2 \sin\theta \; dr \; d\theta = \frac{2a^3}{3},$$

where $R$ is the region bounded by the semi circle $r = 2a\cos\theta$, above the initial line. [25%]

(b) State the Stokes's theorem.

Use Stokes's theorem to evaluate: $\int_S (Curl\, \vec{F}).\boldsymbol{n}\, dS$,

where $\vec{F} = (2x - y)\boldsymbol{i} - yz^2\boldsymbol{j} - y^2z\boldsymbol{k}$ over the upper half sphere of $x^2 + y^2 + z^2 = 1$, bounded by its projection on the XY-plane. [75%]

4. (a) Find $I = \int_V dV$, where $V$ is the volume of the sector of the sphere defined by $0 \le r \le a$, $0 \le \theta \le \alpha$. Evaluate $\int_V \vec{r}\, dV$, over the same sector, and hence show that, if the sphere is of uniform density $\rho$, position vector of its center of mass is given by $\frac{3a}{8}(1 + \cos\alpha)\boldsymbol{k}$.

[40%]

(b) State the Divergence theorem.

Use Divergence theorem to show that $\int_S \vec{F}.\boldsymbol{n}\, dS = \frac{12}{5}\pi a^5$,

where $\vec{F} = x^3\boldsymbol{i} + y^3\boldsymbol{j} + z^3\boldsymbol{k}$ and $S$ is a sphere of radius $a$. [60%]

5. (a) Find the image of the rectangular region in $z$- plane bounded by straight lines $x = -3$, $x = -2, y = 1, y = 2$ under the mapping $w = \frac{1}{z}$. [30%]

(b) Show that the series

$$\sum_{n=0}^{\infty} \frac{(2+i)^n}{(z+i)^n(n+i)^2}$$ is divergent in the region $|z+i| < \sqrt{5}$. [25%]

(c) Evaluate:

$$\int_C \frac{z-1}{(z+1)^2(z-2)} dz$$ , where $C$ is the circle $|z-i| = 2$. [25%]

(d) Find the Taylor series expansion of

$f(z) = \frac{1}{z^2+4}$ about $z = -i$. Find the region of convergence. [20%]

6. The components of a moment of inertia tensor are given by

$$(T_{ij}) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

(a) Find the eigenvalues and the principle axes of the tensor. [25%]

(b) Express the tensor with respect to its principle axes. [30%]

(c) Write the equation of the transformation relating to the coordinates $x_i$ determined by the principle axes. [25%]

(d) Verify the results in (b) by using the transformation law for second order tensor. [20%]

7. (a) Using the variable separable method, solve the following boundary value problem.

$u_{tt} = C^2 u_{xx}$ in $0 < x < L$ , $t > 0$, subject to the boundary conditions:

$u(0,t) = u(L,t) = 0$ , $t > 0$ and initial conditions for $0 < x < L$ are

$$u(x,0) = \begin{cases} \dfrac{3hx}{2L} & 0 < x < \dfrac{2L}{3} \\ \dfrac{3h(L-x)}{L} & \dfrac{2L}{3} < x < L \end{cases}$$

and $u_t(x,0) = 0$ [60%]

(b) Apply fourier sine transformation to solve:

$$\frac{\partial v}{\partial t} = 2\frac{\partial^2 v}{\partial x^2} \quad for\ x \geq 0\ , t \geq 0$$

under the conditions: $u(0,t) = 0$ and $u(x,0) = e^{-x}$. [40%]

8. A paper manufacturer makes grocery bags. He is interested in increasing the tensile strength of their product. It is thought that strength is a function of the hardwood concentration in the pulp. An investigation is carried out to compare three levels of hardwood concentration. Six test specimens are made at each level and all 18 specimens are then tested in random order. The results are shown below:

| Specimen | Tensile Strength (psi) | | |
|---|---|---|---|
| | Level 1 | Level 2 | Level 3 |
| 1 | 12 | 14 | 19 |
| 2 | 17 | 18 | 25 |
| 3 | 13 | 19 | 22 |
| 4 | 18 | 17 | 23 |
| 5 | 19 | 16 | 18 |
| 6 | 15 | 18 | 20 |

(a) Test at 5% level of significance whether the mean tensile strength is same for three levels of hardwood concentration. [85%]

(b) State the assumptions you made. [15%]

9. A company manufactures three products P, Q and R. Each product has to undergo operations on three types of machines $M_1$, $M_2$ and $M_3$ before they are ready for sale. The limit that each product requires on each machine, and the total time per day available on each machine are given in the following table. The table also shows the net profit per unit on the sale of the three products.

(a) Formulate a mathematical model for this problem to maximize the total net profit of the company per day

(b) Solve the model by using the simplex method. [100%]

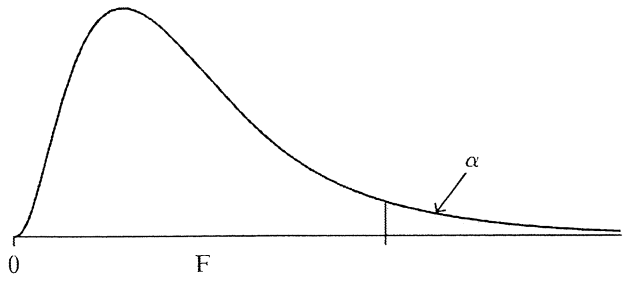
| | Time per unit in minutes | | | Total time available per day in minutes |
|---|---|---|---|---|
| Product / Machine | P | Q | R | |
| $M_1$ | 1 | 2 | 1 | 480 |
| $M_2$ | 2 | 1 | 0 | 540 |
| $M_3$ | 1 | 0 | 3 | 510 |
| Profit per unit in Rs | 4 | 3 | 5 | |

- *END* -

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## TABLE D: *F* Distribution

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$\alpha = .05$

| | $df_1$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $df_2$ | 1 | 2 | 3 | 4 | 5 | 6 | 8 | 12 | 24 | $\infty$ |
| 1 | 161.4 | 199.5 | 215.7 | 224.6 | 230.2 | 234.0 | 238.9 | 243.9 | 249.0 | 254.3 |
| 2 | 18.51 | 19.00 | 19.16 | 19.25 | 19.30 | 19.33 | 19.37 | 19.41 | 19.45 | 19.50 |
| 3 | 10.13 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.84 | 8.74 | 8.64 | 8.53 |
| 4 | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.04 | 5.91 | 5.77 | 5.63 |
| 5 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.82 | 4.68 | 4.53 | 4.36 |
| 6 | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.15 | 4.00 | 3.84 | 3.67 |
| 7 | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.73 | 3.57 | 3.41 | 3.23 |
| 8 | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.44 | 3.28 | 3.12 | 2.93 |
| 9 | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.23 | 3.07 | 2.90 | 2.71 |
| 10 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.07 | 2.91 | 2.74 | 2.54 |
| 11 | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 2.95 | 2.79 | 2.61 | 2.40 |
| 12 | 4.75 | 3.88 | 3.49 | 3.26 | 3.11 | 3.00 | 2.85 | 2.69 | 2.50 | 2.30 |
| 13 | 4.67 | 3.80 | 3.41 | 3.18 | 3.02 | 2.92 | 2.77 | 2.60 | 2.42 | 2.21 |
| 14 | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.70 | 2.53 | 2.35 | 2.13 |
| 15 | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.64 | 2.48 | 2.29 | 2.07 |
| 16 | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.59 | 2.42 | 2.24 | 2.01 |
| 17 | 4.45 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.55 | 2.38 | 2.19 | 1.96 |
| 18 | 4.41 | 3.55 | 3.16 | 2.93 | 2.77 | 2.66 | 2.51 | 2.34 | 2.15 | 1.92 |
| 19 | 4.38 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.48 | 2.31 | 2.11 | 1.88 |
| 20 | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.45 | 2.28 | 2.08 | 1.84 |
| 21 | 4.32 | 3.47 | 3.07 | 2.84 | 2.68 | 2.57 | 2.42 | 2.25 | 2.05 | 1.81 |
| 22 | 4.30 | 3.44 | 3.05 | 2.82 | 2.66 | 2.55 | 2.40 | 2.23 | 2.03 | 1.78 |
| 23 | 4.28 | 3.42 | 3.03 | 2.80 | 2.64 | 2.53 | 2.38 | 2.20 | 2.00 | 1.76 |
| 24 | 4.26 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.36 | 2.18 | 1.98 | 1.73 |
| 25 | 4.24 | 3.38 | 2.99 | 2.76 | 2.60 | 2.49 | 2.34 | 2.16 | 1.96 | 1.71 |
| 26 | 4.22 | 3.37 | 2.98 | 2.74 | 2.59 | 2.47 | 2.32 | 2.15 | 1.95 | 1.69 |
| 27 | 4.21 | 3.35 | 2.96 | 2.73 | 2.57 | 2.46 | 2.30 | 2.13 | 1.93 | 1.67 |
| 28 | 4.20 | 3.34 | 2.95 | 2.71 | 2.56 | 2.44 | 2.29 | 2.12 | 1.91 | 1.65 |
| 29 | 4.18 | 3.33 | 2.93 | 2.70 | 2.54 | 2.43 | 2.28 | 2.10 | 1.90 | 1.64 |
| 30 | 4.17 | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.27 | 2.09 | 1.89 | 1.62 |
| 40 | 4.08 | 3.23 | 2.84 | 2.61 | 2.45 | 2.34 | 2.18 | 2.00 | 1.79 | 1.51 |
| 60 | 4.00 | 3.15 | 2.76 | 2.52 | 2.37 | 2.25 | 2.10 | 1.92 | 1.70 | 1.39 |
| 120 | 3.92 | 3.07 | 2.68 | 2.45 | 2.29 | 2.17 | 2.02 | 1.83 | 1.61 | 1.25 |
| $\infty$ | 3.84 | 2.99 | 2.60 | 2.37 | 2.21 | 2.09 | 1.94 | 1.75 | 1.52 | 1.00 |

Source: From Table V of R. A. Fisher and F. Yates, *Statistical Tables for Biological, Agricultural and Medical Research*, published by Longman Group Ltd., London, 1974. (Previously published by Oliver & Boyd, Edinburgh.) Reprinted by permission of the authors and publishers.