00265

The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc/ B. Ed Degree Programme

Department	: Mathematics
Level	: 04
Name of the Examination	: Final Examination
Course Code and Title	: PEU4302- Linear Algebra
Academic Year	: 2020/2021
Date	: 07.12.2021
Time	: 9.30 a.m.-11.30 a.m.
Duration	: 2 Hours

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of 06 questions in 03 pages.
3. Answer any 04 questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Draw fully labelled diagrams where necessary
5. Relevant log tables are provided where necessary.
6. Having any unauthorized documents/ mobile phones in your possession is a punishable offense.
7. Use blue or black ink to answer the questions.
8. Circle the number of the questions you answered in the front cover of your answer script.
9. Clearly state your index number in your answer script.

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1. (a) Using properties of determinants, solve for x: $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0.$

(b) Show that $\begin{vmatrix} x^3 & x^2 & x & 1 \\ \alpha^3 & \alpha^2 & \alpha & 1 \\ \beta^3 & \beta^2 & \beta & 1 \\ \gamma^3 & \gamma^2 & \gamma & 1 \end{vmatrix} = (x-\alpha)(x-\beta)(x-\gamma)(\alpha-\beta)(\beta-\gamma)(\alpha-\gamma)$.

(c) Find a, b, and c for which the matrix $\begin{pmatrix} a & 1/\sqrt{2} & -1/\sqrt{2} \\ b & 1/\sqrt{6} & 1/\sqrt{6} \\ c & 1/\sqrt{3} & 1/\sqrt{3} \end{pmatrix}$ is orthogonal.

(d) Find the matrix A satisfying the following matrix equation.

$$\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} A \begin{pmatrix} -3 & 2 \\ 5 & -3 \end{pmatrix} = \begin{pmatrix} -2 & 4 \\ 3 & -1 \end{pmatrix}.$$

2. (a) Let $A = \begin{pmatrix} 1 & a & b & a \\ a & b & b & 1 \\ 1 & 1 & 1 & a \end{pmatrix}$. Find the echelon form of A where a and b are distinct non-zero real numbers.

(b) Find the inverse of the following matrix:

$$\begin{pmatrix} 2 & 4 & 3 & 2 \\ 3 & 6 & 5 & 2 \\ 2 & 5 & 2 & -3 \\ 4 & 5 & 14 & 14 \end{pmatrix}$$

(c) Reduce the following matrix to the normal form and hence find its rank:

$$\begin{pmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}$$

3. (a) Solve the following system of homogeneous equations if possible:

$$\begin{aligned} x_1 + 2x_2 + 4x_3 + x_4 &= 4 \\ 2x_1 \quad - \quad x_3 - 3x_4 &= 4 \\ x_1 - 2x_2 - x_3 \quad\quad &= 0 \\ 3x_1 + x_2 - x_3 - 5x_4 &= 5 \end{aligned}$$

(b) For what values of α will the system of equations

$$\begin{aligned} x + y + z &= 1 \\ x + 2y + 4z &= \alpha \\ x + 4y + 10z &= \alpha^2 \end{aligned}$$

have a solution and solve the system completely in each case.

(c) Use Cayley-Hamilton theorem to express $2A^5 - 3A^4 + A^2 - 4I$ as a linear polynomial in A, where:

$$A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}.$$

4. (a) Diagonalize the following matrix if possible.

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ -3 & 5 & 2 \end{pmatrix}$$

(b) Transform the following quadratic form to canonical form by an orthogonal transformation:

$$Q = x_1^2 - x_3^2 - 4x_1x_2 + 4x_2x_3$$

5. Solve each of the following systems of equations by LU – decomposition:

(a) $5x - 5y - 15z = 40$
$4x - 2y - 6z = 19$
$3x - 6y - 17z = 41$

(b) $-x + z = 5$
$2x + 3y - 2z + 6t = -1$
$-y + 2z = 3$
$z + 5t = 7$

6. (a) Let $A = \begin{pmatrix} a & b+ic \\ b-ic & k \end{pmatrix}$: where a, b and k are real constants.

(i) Show that A is a Hermitian matrix.

(ii) Find the eigen values of A.

(b) Let $B = \begin{bmatrix} 2i & 3i \\ 3i & 0 \end{bmatrix}$.

(i) Show that B is a Skew –Hermitian matrix.

(ii) Find the eigen vectors of B.