060

The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc/ B. Ed Degree Programme

Department	**: Mathematics**
Level	**: 04**
Name of the Examination	**: Final Examination**
Course Title and - Code	**: Continuous Functions – PEU4315**
Academic Year	**: 2020/21**
Date	**: 20.12.2021**
Time	**: 1.30 p.m. To 3.30 p.m.**
Duration	**: Two Hours.**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **(6)** questions in **(2)** pages.
3. Answer any **(4)** questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Draw fully labelled diagrams where necessary
6. Involvement in any activity that is considered as an exam offense will lead to punishment
7. Use blue or black ink to answer the questions.
8. Clearly state your index number in your answer script

1.(a) Find the limit points of each of the following subsets A of $\mathbb{R}$

(i)A= $\{2019,2020,2021\}$

(ii)A= $(1,2]$

(iii) $A = \left\{\frac{n}{n+1} : n \in \mathbb{N}\right\}$.

(IV) A=$\left\{\frac{1}{n} : n \in \mathbb{N}\right\} \cup \left\{1 + \frac{1}{n} : n \in \mathbb{N}\right\}$

(b) State whether the following statement is true or false:

Every infinite subset of $\mathbb{R}$ has a limit point.

Justify your answer.

2.(a) Let $f(x) = x^3 + 2x + 7, x \epsilon \mathbb{R}$.

Using the $\varepsilon - \delta$ definition, show that $\lim_{x\to 1} f(x) = 10$.

(b)Let $g(x) = \begin{cases} 1 & if\ x \in (0,1] \\ 0 & if\ \ x = 0 \end{cases}$

Using the $\varepsilon - \delta$ definition, prove that $\lim_{x\to 0^+} g(x) = 1$.

3. (a)Let $a \in \mathbb{R}$ and $f(x) = \begin{cases} 1 + x^2 & ; x \in [0,1) \\ a & ; x = 1 \\ \frac{1}{2}(6 - ax^2) & ; \ x \in (1,2] \end{cases}$

(1) Prove that $\lim_{x\to 1^-} f(x) = 2$ and find $\lim_{x\to 1^+} f(x)$

(ii) Find the value of a such that f is continuous at 1.

(b) Let $f(x) = \frac{1}{x^2}$ for $x \neq 0$. Prove that $\lim_{x\to o} \frac{1}{x^2} = \infty$.

4.(a) Define $f: \mathbb{R} \to \mathbb{R}$ by

$$f(x) = \begin{cases} x & ; x \in \mathbb{Q} \\ -x & ; x \in \mathbb{Q}^c \end{cases}$$

Prove that $\lim_{x \to 0} f(x) = 0$.

(b) Let $g(x) = \frac{x+1}{3x+2}$ for $x > 0$. Prove that $\lim_{x \to \infty} g(x) = \frac{1}{3}$.

5.(a) Let $f(x) = x^3 + 2x - 5, x \in \mathbb{R}$. Using the $\varepsilon - \delta$ definition prove that f is Continuous at 1.

(b) Suppose f is a function defined on $I = (a, b)$ and let $c \in (a, b)$. Prove that

If f is continuous at , then $|f|$ is also continuous at c.

Is the inverse true? Justify your answer.

6.(a) State the definition of a continuous function on a closed interval.

(b) Define $h: [-1, \quad 5] \to \mathbb{R}$ by $h(x) = \sqrt{5 + 4x - x^2}$ for $x \in [-1, \quad 5]$ show that h is continuous on $[-1, \quad 5]$.