00150

The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc/ B. Ed Degree Programme

Department	: Mathematics
Level	: 05
Name of the Examination	: Final Examination
Course Title and - Code	: Combinatorics- PEU5302
Academic Year	: 2020/21
Date	: 14-12-2021
Time	: 01.30 pm - 03.30 pm
Duration	: Two (02) hours

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **Six (06)** questions in **Three (03)** pages.
3. Answer any **Four (04)** questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Draw fully labeled diagrams where necessary
6. Involvement in any activity that is considered as an exam offense will lead to punishment
7. Use blue or black ink to answer the questions.
8. Clearly state your index number in your answer script

01. A boy invited 15 friends to his birthday party but only 10 of them participated.

(a) In how many times will,

(i) 3 best friends always participate in the party?

(ii) at least one of the 3 best friends participate in the party?

(b) In that party, in how many ways can

(i) 15 identical pieces of cake be served among the participated friends?

(ii) 15 identical gift items be shared among those participated friends, if each of them must get at least one item?

(iii) 20 distinct toys be distributed among the participated friends?

(iv) 20 different fruits be equally divided into packets so that each of the participated friends receive one?

02. (a) In how many ways can 12 boys and 8 girls be seated at a round table,

(i) without any restriction?

(ii) if all the boys sit together?

(ii) if no two girls sit together?

(iii) if all the girls do not sit together?

(b) In a game of sports, between two teams with equal number of players, every player shakes hand with every other player. If there was a total of 66 handshakes at the induction event, how many players participated from a team in the game?

(c) 10 busses run between 2 cities. In how many ways can a passenger go in a bus from one city to the other city and returns by a different bus?

(d) In how many ways can a group of 24 students be divided into 3 groups of 4 students or 4 groups of 3 students?

03. (a) Let A, B and C be any 3 events of a Universal set $\cup$. Define the conditional probability $p(A/B)$. Prove each of the following conditions:

(i) $p(A/B) \geq \dfrac{p(A)+p(B)-1}{p(B)}$.

(ii) $p(A/B)+p(A'/B)=1$, where A' is the complement of the event A.

(iii) if $p(A/C) > p(B/C)$ and $p(A/C') > p(B/C')$ then $p(A) > p(B)$, where C' is the complement of the event C.

(b) Two machines are available at a manufacturing process which runs with only one machine operating at a time. Once the 1st machine fails, the 2nd machine will be operating. It is also given that the probability of the 1st machine fails is 0.07 and the probability of the 2nd machine fails, given that the 1st machine has already failed is 0.02. Also, the 2nd machine can fail only if the 1st has already failed. Find the probability of the 2nd machine fails.

(c) In a population of a particular species, the probability that the species lives to at least 10 years is 75% and 50% probability that it will live at least 12 years. If a species is 10 years old now, find the conditional probability that it will survive for another 2 years.

04. By substituting, first few natural numbers, identify the starting natural number of each of following statements in which truth value becomes true. Use the principle of mathematical induction to show that the variant form of each of them is true:

(a) $2^n > n^2$.

(b) $n! > 2^n$.

(c) $4^{n-1} > n^2$.

(d) $(2n)! > 2^n (n!)^2$.

05. Let ${}^{n}C_{r}$ be the number of ways selecting of r objects among n distinct objects, where $0 \le r \le n$.

Use ONLY the combinatorial argument to prove each of the following identities:

(a) ${}^{n}C_{r}\,{}^{r}C_{k} = {}^{n}C_{k}\,{}^{n-k}C_{r-k}$.

(b) ${}^{2n}C_{n} = 2^{2n-1}C_{n-1}$.

(c) ${}^{m+n}C_{r} = \sum_{k=0}^{r} {}^{m}C_{k}\,{}^{n}C_{r-k}$, where $0 \le k \le r \le m$.

Hence, deduce that ${}^{2n}C_{n} = \sum_{k=0}^{n} ({}^{n}C_{k})^{2}$.

06. (a) If the coefficients of 3 consecutive terms in the binomial expansion of $(1+x)^{n}$ are in the ratio $1:2:3$, find the value of n and those three terms.

(b) How many terms of the multinomial expansion of $(w+x+y+z)^{10}$ are of the form $w^{p}x^{q}y^{r}z^{s}$ where $p \ge 3$, $q \ge 2$, $r \ge 1$ and $s \ge 0$.

(c) Find the multinomial coefficient of each of the following terms with respect to the given expansions of the expressions:

(i) x^{3} in the expansion of $(1+x+x^{2}+x^{3})^{10}$.

(ii) x^{3} in the expansion of $(1-2x+3x^{2})^{5}$.

(iii) $w^{4}x^{2}y^{3}z$ in the expansion of $(w-2x+3y-z)^{10}$.

Are the coefficients of $w^{4}x^{2}y^{3}z$ and $w^{4}x^{3}y^{2}z$ same? Justify your answer.