00136

The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc/ B. Ed Degree Programme

Department	**: Mathematics**
Level	**: 05**
Name of the Examination	: Final Examination
Course Title and - Code	**: Introduction to Complex Analysis – PEU5304/PUU3141**
Academic Year	**: 2020/2021**
Date	**: 03.12.2021**
Time	**: 09.30 am – 11.30 am**
Duration	**: 02 hrs**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This paper consists of **TWO** sections, Section **A** and Section **B**. Section **A** is compulsory and it consists of SIX Structured Essay Questions. You should answer in the space under each question.
3. Section B consists of **FIVE** essay type questions and answer only **THREE** of them. Each question in Section B carries 100 marks.
4. This paper consists of 05 pages.
5. Answer for each question in section B should commence from a new page.
6. Involvement in any activity that is considered as an exam offence will lead to punishment
7. Use a blue or black ink pen to answer the questions.
8. Clearly state your index number in your answer script.
9. At the end of the exam, attach Section A to the answer booklet and hand over to the supervisor.

SECTION A

1. Answer all the questions in the space provided.

Prove or disprove the following statements. Justify your answer.

(a) $\mathrm{Re}(z_1 z_2) = \mathrm{Re}(z_1)\mathrm{Re}(z_2)$ where $z_1, z_2 \in \mathbb{C}$.

(b) If $\bar{z} = -z$, then z is pure imaginary.

(c) If $z \neq 0$, then $\mathrm{Arg}(z + \bar{z}) = 0$.

(d) If z is a point in the second quadrant, then $i\bar{z}$ is in the fourth quadrant.

(e) For complex numbers z_1, z_2, if $|z_1| = 12$ and $|z_2 - 3 - 4i| = 5$ then the minimum value of $|z_1 - z_2|$ is 7.

(f) $\mathrm{Im}(e^{i\theta}) = \sin\theta$.

SECTION B

Answer **THREE** Questions **ONLY**.

02. (a) Let z be a complex number. Prove each of the following.

(i) $z=\bar{z}$ holds if and only if $z\in\mathbb{R}$.

(ii) $z\bar{z}=|z|^2$

(b) Prove that if $|z_1|=|z_2|=1$ and $z_1z_2\neq -1$ then $\left(\frac{z_1+z_2}{1+z_1z_2}\right)$ is a real number.

(c) Let z be a complex number with imaginary part 50. Find a positive integer m such that $\left(\frac{z}{z+m}\right)=2i$.

03. (a) (i) State the De Movivre's Theorem for a positive integral index.

(ii) Evaluate the following expression.

$$\left(\frac{\left(\sqrt{3}+i\right)^5\left(1-i\right)^{10}}{\left(-1-i\sqrt{3}\right)^{10}}\right).$$

(b) Let z be a complex number such that $z+\frac{1}{z}=2\cos 3^0$. Find the least integer that is greater than $z^{1000}+\frac{1}{z^{1000}}$.

04. (a) Let z_1, z_2 be non-zero complex numbers. Prove each of the following.

(i) z_1z_2 is a non-zero complex number.

(ii) $\arg(z_1z_2)=\arg(z_1)+\arg(z_2)$.

(b) (i) Express $\left(-1+\sqrt{3}i\right)\left(2-2i\right)$ in polar form.

(ii) Is $\text{Arg}(z_1z_2)=\text{Arg}(z_1)+\text{Arg}(z_2)$ for any $z_1, z_2\in\mathbb{C}\setminus\{0\}$?

(iii) Find all the complex numbers z, such that $|z+3|-|z-3|=0$ and $\arg(-iz)=-\pi+2n\pi, n\in\mathbb{Z}$.

05. (a) Find all the complex roots of the equation $z^6=-9$.

(b) (i) Let $p(z)=0$ be a non-constant complex polynomial with real coefficients. If α is a root of $p(z)=0$ then prove that $\overline{\alpha}$ is also a root of $p(z)=0$.

(ii) Find all real numbers r for which the equation $z^3+(3+i)z^2-3z-(r+i)=0$, has a real root.

06. (a) Find i^i and then find its principal value.

(b) Is $\log i^{\frac{1}{2}}=\dfrac{\log i}{2}$? Justify your answer.

(c) Evaluate $\sin\left(\dfrac{\pi}{4}+i\right)$.

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