THE OPEN UNIVERSITY OF SRI LANKA
Faculty of Engineering Technology
Department of Mathematics & Philosophy of Engineering

Bachelor of Technology Honors in Engineering

Final Examination (2019/2020)
MHJ5334/MPJ5134: The Nature of Mathematics I

Date: 31st July 2020 (Friday) **Time: 1400 hrs. – 1700 hrs.**

Instructions:

- *Answer **five (05)** questions only.*
- *Number of pages in the paper is five (05).*
- *All the symbols are in standard notation unless they are defined.*

Some important notes

- ***Axioms of real numbers***

  Let a, b and c be real numbers

  A1 : $a + b = b + a$

  A2 : $a + (b + c) = (a + b) + c$

  A3 : $a + 0 = a$

  A14 : $a + (-a) = 0$

  M1 : $a.b = b.a$

  M2 : $a.(b.c) = (a.b).c$

  $M3$: $a.1 = a$

  $M4$; $a.\frac{1}{a} = 1$

  D : $a.(b + c) = ab + ac$

- ***Axioms of order***

  There is a subset $\mathbb{P}$ of $\mathbb{R}$ called the positive real numbers satisfying:

  $P1$: ***If a, b in $\mathbb{P}$ then $a + b$ in $\mathbb{P}$***

  $P2$; ***If a, b in $\mathbb{P}$ then $a.b$ in $\mathbb{P}$***

  Exactly one of the following statements:

  $P3$: a ***in*** $\mathbb{P}$, $a = 0, -a$ ***in*** $\mathbb{P}$

- ***Integer a divides integer b $(a|b)$ if and only if $b = ac$ for some integer c.***

- *Mathematical Induction:*

  If we can prove both of the following steps, we then know that the proposition is a result (Theorem), true for all n

  (I) *The initial step:*

  Prove the proposition is true for $n = 1$

  (II) *Inductive step:*

  Assume the proposition is true fo $n = k$, *and on this basis prove it is true for* $n = k + 1$

- *Results of sets.*

  A, B *and* C *be any three subsets of the universal set* E.

  Union $(\cup)$

  $U(1): A \cup A = A$

  $U(2): A \cup B = B \cup A.$

  $U(3): A \cup \emptyset = A$

  $U(4): A \cup (B \cup C) = (A \cup B) \cup C$

  $U(5): A \cup B = B \Leftrightarrow A \subseteq B.$

  Intersection $(\cap)$

  $Int(1): A \cap A = A$

  $Int(2): A \cap B = B \cap A$

  $Int(3): A \cap \emptyset = \emptyset$

  $Int(4): A \cap (B \cap C) = (A \cap B) \cap C$

  $Int(5): A \cap B = A \Leftrightarrow A \subseteq B$

  Distribution

  $Dist(1)\ A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

  $Dist(1)\ A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

  Compliment

  $C(1): (A^C)^C = A$

  $C(2): (A \cup B)^C = A^C \cap B^C$

  $C(3): (A \cap B)^C = A^C \cup B^C$

  $C(4): A \cap A^C = \emptyset$

  $C(5): A \cup A^C = E$

  $C(6): A \subseteq B \Rightarrow B^C \subseteq A^C$

  $C(7): A \backslash B = A \cap B^C$

  $C(8): A \Delta B = (A \cap B^C) \cup (A^C \cap B)$

End of the notes

Q1 Let $a, b \in \mathbb{R}$. Using the axioms of real numbers, prove that:

(a) i) If $a, b, c \in \mathbb{R}$ and $a + b = a + c$, then $b = c$. (20%)

ii) $a.0 = 0$ (15%)

iii) If $a, b \in \mathbb{R}$ and $a.b = 0$, then either $a = 0$ or $b = 0$. (15%)

(b) Using the axioms of order, prove that if $a > 0$, then $\frac{1}{a} > 0$. (25%)

(c) Using principle of mathematical induction, prove that for all positive integer n, $1.1! + 2.2! + 3.3! + \cdots + n.n! = (n + 1)! - 1$. (25%)

Q2. (a) Using truth tables show that:

i) $\sim(p \Rightarrow q) \equiv p \wedge \sim q$. (15%)

ii) $p \Rightarrow (q \wedge r) \equiv (p \Rightarrow q) \wedge (p \Rightarrow r)$. (15%)

iii) $(p \Rightarrow q) \Rightarrow (\sim q \Rightarrow \sim p)$ is a tautology. (15%)

iv) $\sim[(p \wedge (p \Leftrightarrow q)) \Rightarrow q]$ is a contradiction. (15%)

(b) Consider the following argument:

If I study Logic, then I will get A pass. If I do not go to a party, then I will study Logic. But I did not get A pass. Therefore, I went to the party.

i). Express the above argument in symbolic form with following notations. p: I study Logic. q: I will get A pass. r: I go to a party. (10%)

ii) Determine the validity of the above argument. (10%)

(c) Determine whether the collection of propositions $p \Rightarrow q$, $\sim p \vee \sim q$, $\sim(q \Rightarrow p)$ is consistent or inconsistent. (20%)

Q3. (a) Using direct proof method, prove that the following.

i) If a even and b is odd, prove that $ab + 5$ is odd. (15%)

ii) If n is an integer, then $1 + (-1)^n(2n - 1)$ is divisible by 4. (30%)

(b) For all $x, y \in \mathbb{R}$, show that the following:

i) Assuming that $|xy| = |x||y|$, deduce that $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$ if $y \neq 0$. (10%)

ii) $|x - y| > 0$ if and only if $x \neq y$, (20%)

(c) Find the set of all solutions of the inequality $|x - 4| < 8 - |x + 2|$, and draw the related graph, where x is a real number. (25%)

Q4 (a) Using algebraic law of propositions, $\sim\sim p \equiv p$ and $p \Rightarrow q \equiv \sim p \vee q$, prove that

i) $\sim(p \Rightarrow q) \equiv p \wedge \sim q$ (ii) $\sim(p \wedge q) \equiv p \Rightarrow \sim q$ (20%)

(b) Consider the following statement. φ : For all x and for all y in the real numbers, $(x+y)^2$ is greater than $(x-y)^2$ implies that xy is greater than zero

i) Write down φ using quantifiers and symbols. (15%)

ii) Deduce the symbolic form of $\sim \varphi$. (15%)

iii) Prove that φ is true. (10%)

(c) Consider the statement. ψ : There exist x and y in the real numbers such that $(x+y)^2$ is less than $(x-y)^2$ and xy is greater than or equal to zero.

i) Write down ψ using quantifiers and symbols. (15%)

ii) Deduce the symbolic form of $\sim\psi$. (15%)

iii) Prove that ψ is false. (10%)

Q5 Let A, B and C be three subsets of the universal set E.

(a) Give examples to show that the following statements are not generally true.

i) $A \cup B = A \cup C \Rightarrow B = C$. ii) $A \cap B = A \cap C \Rightarrow B = C$. (20%)

(b) Using Venn-diagram method answer the following.

i) If $A \subseteq B$, then find

(α) $A \backslash B$ (β) $A \cap B$ (γ) $A \cup B$ (15%)

ii) $A \cap B = A \backslash (A \backslash B)$ (15%)

iii) $A \backslash B = B \backslash A$ if and only if $A = B$. (15%)

iv) $(A \cap B^C) \cup (A^C \cap B) = (A \cup B) \cap (A \cap B)^C$ (35%)

Q6 Let A, B and C be three sub sets of the universal set E

(a) Using the results of sets, prove that the following

i) $A\backslash(A \cup B) = (A\backslash B) \cap (A\backslash C)$ (15%)

ii) $A \cap (B\backslash C) = (A \cap B)\backslash(A \cap C)$ (15%)

iii) $A \cap E = A.$ (10%)

iv) $A \cup B = B \Leftrightarrow A \cap B^C = \emptyset$ (20%)

(b) Using the logic method, prove that

i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (20%)

ii) $A \subseteq B \Leftrightarrow A = A \cap B$ (20%)

Q7 (a) Define the product $A \times B$ of the sets A and B. Prove that (05%)

i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$ (15%)

ii) $A \times (B\backslash C) = (A \times B)\backslash(A \times C)$ (15%)

(b) Let $A = \{1, 2, 3, 4\}$. Define a relation ρ on A and for any $x, y \in A$ $x\rho y \Leftrightarrow$ $x + y$ is even

i) Write down $A \times A$ and the set R and for any $x, y \in A$, $R =$ $\{(x, y) | x\rho y\}$. State whether $A \times A = R$ or not. Justify your answer. (25%)

ii) Express ρ^{-1} as a set of ordered pairs. (10%)

iii) Show that ρ is reflexive, symmetric, and transitive. (15%)

(c) Define a relation ρ on $\mathbb{R}$ and for any $x, y \in \mathbb{R}$, $x\rho y \Leftrightarrow x \leq y^3$. Show that ρ is not reflexive, not symmetric, and not transitive. (15%)

Q8 (a) Define Greatest common divisor of two integers (10%)

i) Let a and b be two integers. Show that there exist two integers x and y such that $ax + by = 1$, then a and b are relatively prime. (10%)

ii) Let a, b and c be integers such that $a|b$ and $b|c$. If a and b are relatively prime show that $ab|c$. (20%)

(b) Using well ordering principle of $\mathbb{N}$ and assuming $\sqrt{2} < \sqrt{3}$, prove that

$$\left(\frac{\sqrt{2} + \sqrt{3}}{2}\right)^n \leq \frac{(\sqrt{2})^n + (\sqrt{3})^n}{2}.$$ (60%)

End

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